Advanced Instability Analysis

Dipl.-Ing. R. Blumenthal · Dipl.-Ing. S. Bomberg · T. Stanko, M.Sc. · Prof. Wolfgang Polifke, Ph.D.

I Background and Motivation

Thermoacoustic instabilities are a major concern in combustion systems, such as gas turbines or rocket engines.

It was shown that operators describing the dynamics of thermoacoustic systems generally are non-normal, i.e. their eigenvectors/eigenfunctions are not orthogonal [2] . As a consequence, certain effects are present in such systems that cannot be captured by means of the classical stability analysis. Due to non-normality, the amount of fluctuation energy can temporarily increase, even for a linear, asymptotically stable case. It is argued that this transient growth can lead to fluctuation amplitudes large enough to no longer allow for the assumption of linearity [3]. In that case, nonlinear effects must be taken into account which may take the system to a different asymptotic state: Figure I shows the development of fluctuation energy in a thermoacoustic system over time. For the linear case, after an initial transient growth, all oscillations decay. If nonlinearities are accounted for, transient growth triggers the system into a limit cycle. In general, the strength of transient growth is heavily depending on the initial conditions and the definition of (a norm of) fluctuation energy.

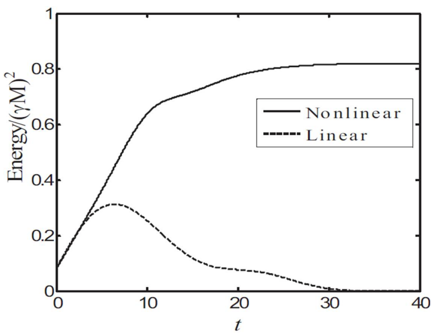

Figure 1: Transient growth and triggering.

II Project Goals: Pertinent Questions

As pointed out the effects of nonlinearity and non-normality may render the classical stability analysis inadequate. Thus, a toolbox for an advanced instability analysis is required. A number of topics shall be studied:

- Under which circumstances does non-normality trigger the system into a nonlinear instability regime?
- Does there exist a universal transitional mechanism?
- Which initial conditions lead to maximum growth? Are these initial conditions physically attainable?
- By which means does inter-modal energy transfer occur?
- Which model structures and identification schemes are suitable for describing such systems?

III Project Summary

The effects of non-normality (in combination with nonlinearity) on thermoacoustic systems shall be studied for the case of conical and v-shaped, laminar premix flames. Numerical simulations are expected to yield information about appropriate identification schemes and model structures for reduced models. Consequently, tools shall be developed that allow to perform an advanced stability analysis for systems of an applied interest with a reasonable amount of computational effort and time.

A so-called monolithic numerical solver is to be implemented. The fully compressible Navier-Stokes Equations are solved including flow-combustion-acoustics coupling. This solver is intended to generate reliable reference data as well as to study the impact of various parameters on non-normality and the related system identifiability.

In a second approach, a less computationally involving solver is to be created. Assuming low Mach number and compact flames, asymptotic analysis allows to divide the numerical domain into an incompressible hydrodynamic region and an acoustic region [1]. This procedure is expected to drastically reduce the computational effort, so that this solver may serve as a starting point for the analysis of more complex systems.

The third approach consists of finding reduced-order models, which are capable of capturing the dominant non-normal and nonlinear effects. System identification and model reduction techniques as well as bifurcation analysis shall be performed to develop new numerical tools.

The numerical studies shall be complemented by – and validated against – experimental data obtained at the Indian Institute of Technology in Madras, India. Furthermore, a comparison to results from other existing methods (e.g. Flame Describing Function technique) shall be carried out.

References

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With the support of the Technische Universität München – Institute for Advanced Study, funded by the German Excellence Initiative.