

The Society shall not be responsible for statements or opinions advanced in papers or in discussion at meetings of the Society or of its Divisions or Sections, or printed in its publications. Discussion is printed only if the paper is published in an ASME Journal. Papers are available from ASME for fifteen months after the meeting.
Printed in USA.

Film Cooling and Heat Transfer in Nozzles

J. STOLL and J. STRAUB

Lehrstuhl A für Thermodynamik, Technische Universität München
München, FRG

ABSTRACT

In this paper experimental and theoretical investigations on heat transfer and cooling film stability in a convergent-divergent nozzle are presented.

Compressed air is injected into hot air in the inlet region of the nozzle and the influence of the strong favourable pressure gradient in the nozzle on turbulent heat transfer and mixing is examined.

The experiments cover measurements of wall pressures, wall temperature and wall heat flux.

Calculations with a parabolic finite difference boundary layer code have been performed using a well known k, ϵ -turbulence model with an extension paying regard to acceleration. As a result the calculated wall heat flux is compared with the measured heat flux.

NOMENCLATURE

c_p	specific heat at constant pressure
C_μ, C_1, C_2, C_3	turbulence model constants
f_μ, f_1, f_2, f_3	turbulence model wall functions
f	wall function $f = \min\left(\frac{R_t}{3.72 R_k}, 1.\right)$
H	total enthalpy
h	specific enthalpy
\mathbf{I}	Unity tensor
k	turbulent kinetic energy
K	acceleration parameter $K = -\frac{\mu}{\rho^2 u_\infty^2} \frac{\partial p}{\partial x}$
l	turbulent mixing length
\dot{m}	mass flow
p	pressure
Pr_t	turbulent Prandtl number $Pr_t = \frac{\mu_t c_p}{\lambda_t}$
R	gas constant

R_k	turbulence Reynolds - number = $\frac{k^2 \rho}{\epsilon \mu}$
R_t	turbulence Reynolds - number = $\frac{\epsilon \mu}{\sqrt{k} y \rho}$
R_l	turbulence Reynolds - number = $\frac{\mu}{\sqrt{k} l \rho}$
s	slot height
T	temperature
u	mean velocity in streamwise direction
u_τ	friction velocity = $\frac{\mu_w}{\rho_w} \frac{\partial u}{\partial y}$
u'', v'', w''	fluctuating velocities in x,y,z direction
\mathbf{w}	velocity vector in general coordinates
x, y, z	quasiorthogonal streamwise coordinates
x	streamwise direction; distance from slot
y	cross - stream coordinate; distance from wall
z	coordinate parallel to the wall
x_t	distance from turbulence grid
Δy	stream tube width
Δx^+	dimensionless streamwise step = $\Delta x u_\tau \frac{\rho_w}{\mu_w}$
y^+	dimensionless wall distance = $y u_\tau \frac{\rho_w}{\mu_w}$
δ	boundary layer thickness
ϵ	turbulent dissipation rate = $\frac{1}{\rho} \tau'' : \text{Grad } \mathbf{w}''$
λ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity $\nu = \frac{\mu}{\rho}$
ρ	density
σ_ϵ	turbulent diffusion Prandtl - number for turbulent dissipation rate
τ	shear stress
$\boldsymbol{\tau}$	stress - tensor
	$\boldsymbol{\tau} = \mu \{ \text{Grad } \mathbf{w} + (\text{Grad } \mathbf{w})^T - \frac{2}{3} \text{div}(\mathbf{w})\mathbf{I} \}$

Subscripts and Superscripts

-	denotes time averaged value
\sim	denotes mass averaged mean value
"	denotes fluctuating part of variable
o	denotes stagnation condition
c	coolant
w	wall
∞	free stream
r	recovery condition
t	turbulent

INTRODUCTION

Some authors have already worked on heat transfer in nozzles, e.g. (Back et.al. 1964, 1970, 1972), (Boldmar et.al. 1967, 1972), (Winkler, Grigull 1977), (Bauer et.al. 1977, 1978) and an increasing number of investigations has been undertaken on heat transfer in accelerated compressible flow e.g. (Kays et.al. 1970), (Blair 1982), (Wang et.al. 1985), (Rued, Wittig 1985).

Much fundamental research has been done considering the injection geometry and the stability of cooling films. Many publications have dealt with the comparison of adiabatic and isothermic wall experiments trying to generalize the results using scaling laws (Jones, Forth 1986) and trying to generalize the results using dimensionless variables based on recovery temperatures (Ligrani, Camci 1985) and giving density ratio and variable property corrections in the case of compressible flow.

Only a few authors have examined the influence of mainstream pressure gradients on heat transfer to film cooled surfaces (Hay et.al. 1985). Heat transfer to a film cooled nozzle wall is as well influenced by the strong favorable pressure gradient as by large density ratios. Furthermore the recovery temperature of both the hot gas and the coolant are submitted to substantial changes in the streamwise direction. Therefore a presentation of our test results in non-dimensional form would not allow for generalization.

In order to generalize results, it is important to test and improve existing computer codes in comparison with experimental data. The work on comparison of finite difference codes for prediction of heat transfer to film cooled surfaces (Crawford 1986) is still continued. In the present investigation we compare calculations with a low Reynolds number k, ϵ -model (Lam, Bremhorst 1981) modified for accelerated flows to our experiments.

EXPERIMENTS

EXPERIMENTAL SET - UP

The experimental investigations have been performed in the wind tunnel of our institute. Compressed air is used both as hot medium and as coolant.

The hot air is compressed in a two-stage radial compressor to a maximum stagnation pressure of 0.28 MPa and a

maximum stagnation temperature of 485 K. The hot air passes a mixer, a straightener and a transition pipe where the cross section is changed from a round to a rectangular one of nearly the same area before it enters the test section with 2.8 kg/s maximum mass flow rate.

The cooling air is compressed in a smaller two stage compressor with intermediate heat exchanger. It enters the test section with a stagnation temperature of approximately 305 K and a maximum mass flow rate of 0.25 kg/s.

The whole apparatus is driven in steady state.

TEST SECTION

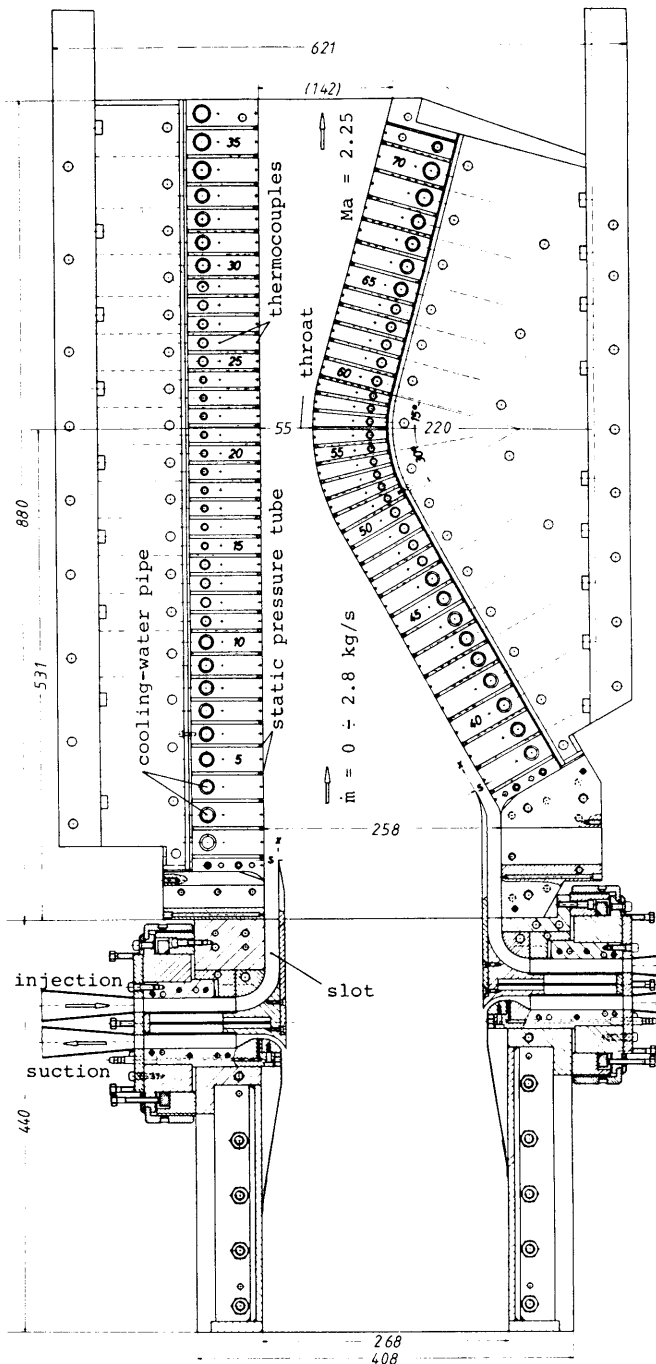


Fig.1 Test section

The test section is an unsymmetrical rectangular water-cooled CD-nozzle (fig.1) which is designed for a normal shock at its exit in the maximum loading case of the compressor allowing a maximum outlet Mach-number of 2.25. The geometry of the rectangular test section in the x,y plane is shown in fig.1. The depth normal to fig.1 is 99 mm (fig.3) avoiding serious 3D-effects. The rectangular 30° - 15° semiangle convergent-divergent nozzle is constructed with a plane and a contoured wall. Thus it is possible to examine the influence of the favourable pressure gradient in the absence of effects of wall curvature.

The nozzle was built up with 35 separately water-cooled segments (fig.3) of copper on each of the two side walls to allow very accurate measurements of the heat flux from the air to the nozzle walls. By adjusting the cooling water flow rate for each segment it is possible to establish a defined thermal boundary condition. The segments are well isolated to each other by air and a thin stainless steel tube at the nozzle surface. The gas side nozzle walls have been chrome plated in order to avoid oxidation and to reduce thermal radiation. The average surface roughness has been determined as $7.3 \pm 1.3 \mu\text{m}$.

The mass flux ratio in the case of the film cooling experiments is defined by varying the hot air mass flow and the cooling air mass flow for different slot heights for the cooling film. The inlet region of the nozzle with inlet geometry is shown in fig.2.

MEASUREMENT TECHNIQUES

The experimental investigation covers measurements of wall temperature, wall heat flux, wall pressures, stagnation pressure and temperature and measurements of the mass flow rates. Wall temperature and wall heat flux were measured with the heat flux meters as shown in fig.2 using two thermocouples installed in the copper segments. In steady state conditions the wall heat flux and the wall temperature are calculated from the onedimensional heat conduction equation. 2D Finite Element Calculations for the copper segments have proved the high accuracy of this simple method. The thermocouples are connected to a scanner and a digital voltmeter.

The wall pressures are taken from the small steel tubes between the copper segments and transferred to a Scanivalve pressure scanning system. Total pressures and total temperatures are taken from combined probes.

The mass flow rate of the hot air is calculated from onedimensional gas dynamics relations in the absence of a cooling film. The mass flow rate of the cold air is measured with a standard orifice. The inlet velocities are calculated from the inlet geometry (figs.2,3). A HP 1000 Minicomputer was used to acquire the data from the pressure scanning system and the digital volt meter during the test run allowing immediate evaluation of the mass flow rates and the distribution of pressure, wall temperature and wall heat flux. Using the computer continuously evaluating the wall temperature and the wall heat flux during scanning it was possible to establish isothermal wall conditions by manually adjusting the cooling water control in an acceptable time.

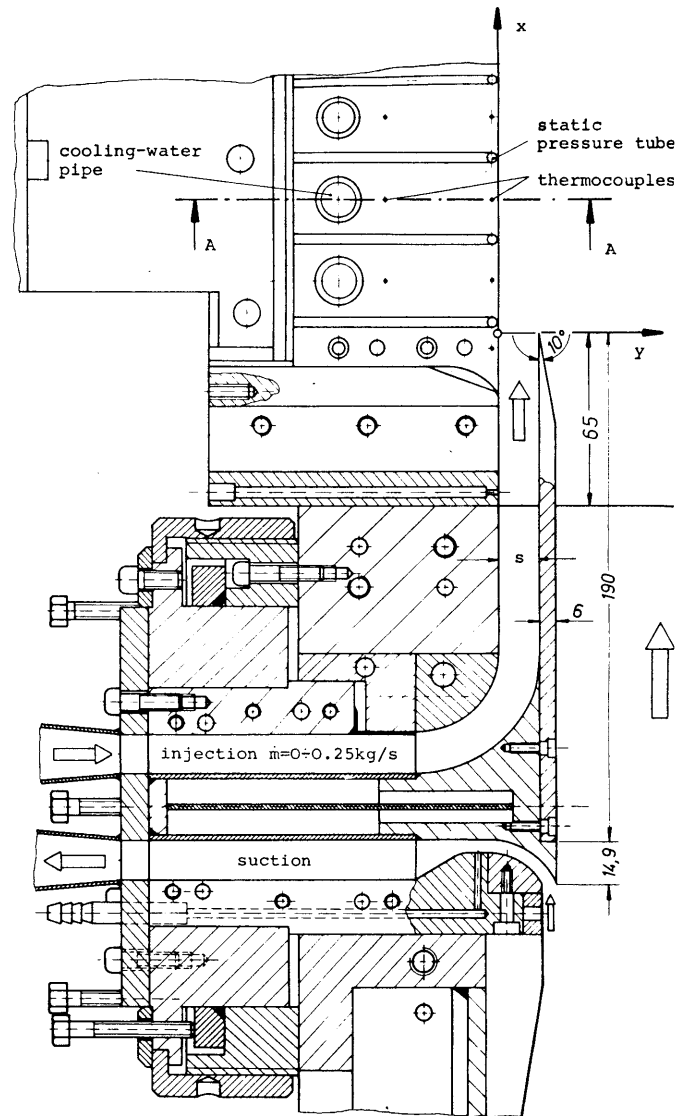


Fig.2 Inlet geometry

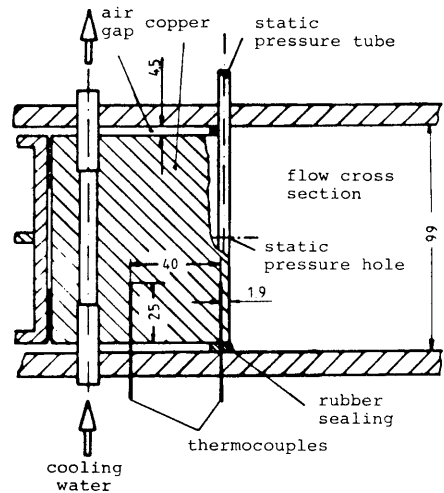


Fig.3 Sectional view; copper segment

CALCULATIONS

MEAN FLOW EQUATIONS

The mean flow equations have been derived from the conservation equations using Favre mass-averaged decomposition thus keeping the correlation terms arising from density fluctuations to a minimum and maintaining almost the same forms than the equations written for incompressible flows (Favre 1965, 1975; Vandrome, Minh 1984).

Additional density fluctuation correlations remain unmodelled in this presentation, they are omitted right from the very beginning. The equations are given in general coordinates for stationary conditions. The mathematical notation for operators and operations in general coordinates is defined in (Klingbeil 1966).

continuity equation

$$\text{div}(\bar{\rho}\tilde{\mathbf{w}}) = 0 \quad (1)$$

momentum equations

$$\text{Div}(\bar{\rho}\tilde{\mathbf{w}}\tilde{\mathbf{w}}) = -\text{grad}\bar{p} + \text{Div}\bar{\boldsymbol{\tau}} + \text{Div}(-\bar{\rho}\tilde{\mathbf{w}}''\mathbf{w}'') \quad (2)$$

total enthalpy equation

$$\text{div}(\bar{\rho}\tilde{\mathbf{w}}\tilde{H}) = \text{div}(\lambda\text{grad}\tilde{T}) + \text{div}(\bar{\boldsymbol{\tau}}\cdot\tilde{\mathbf{w}}) + \text{div}(-\bar{\rho}\tilde{\mathbf{w}}''\mathbf{H}'') \quad (3)$$

$$\tilde{H} = \tilde{h} + \frac{\tilde{\mathbf{w}}\cdot\tilde{\mathbf{w}}}{2} + \frac{\mathbf{w}''\cdot\mathbf{w}''}{2} = \tilde{h} + \frac{\tilde{\mathbf{w}}\cdot\tilde{\mathbf{w}}}{2} + k \quad (4)$$

$$H'' = h'' + \tilde{\mathbf{w}}\cdot\mathbf{w}'' + \frac{\mathbf{w}''\cdot\mathbf{w}''}{2} - \frac{\mathbf{w}''\cdot\mathbf{w}''}{2} \quad (5)$$

Assuming an ideal gas the connection between the variables is given by the ideal gas law

$$\bar{p} = \bar{\rho}R\tilde{T} \quad (6)$$

and the equation of state for the enthalpy

$$d\tilde{h} = c_p d\tilde{T} \quad (7)$$

TURBULENT TRANSPORT

Neglecting turbulent diffusive transport in the streamwise direction the reduced boundary layer equations contain only two important terms $-\bar{\rho}\tilde{\mathbf{u}}''\mathbf{v}''$ and $-\bar{\rho}\tilde{\mathbf{v}}''\mathbf{H}''$. The turbulent stress $-\bar{\rho}\tilde{\mathbf{u}}''\mathbf{v}''$ in the momentum equation and the turbulent total enthalpy flux $-\bar{\rho}\tilde{\mathbf{v}}''\mathbf{H}''$ in the total enthalpy equation are modelled using the eddy viscosity concept

$$-\bar{\rho}\tilde{\mathbf{u}}''\mathbf{v}'' = \mu_t \frac{\partial \mathbf{u}}{\partial y} \quad (8)$$

and after rearranging

$$-\bar{\rho}\tilde{\mathbf{v}}''\mathbf{H}'' = -\bar{\rho}\tilde{\mathbf{v}}''\mathbf{h}'' - \tilde{\mathbf{u}}\bar{\rho}\tilde{\mathbf{u}}''\mathbf{v}'' - \bar{\rho}\tilde{\mathbf{v}}''\mathbf{k}'' \quad (9)$$

the eddy diffusivity concept for the turbulent heat flux

$$-\bar{\rho}\tilde{\mathbf{v}}''\mathbf{h}'' = \lambda_t \frac{\partial \tilde{T}}{\partial y} = \frac{\lambda_t}{c_p} \frac{\partial \tilde{h}}{\partial y} \quad (10)$$

and for the flux of turbulent kinetic energy

$$-\bar{\rho}\tilde{\mathbf{v}}''\mathbf{k}'' = \mu_t \frac{\partial k}{\partial y} \quad (11)$$

The equation for the total enthalpy flux may now be written

$$-\bar{\rho}\tilde{\mathbf{v}}''\mathbf{H}'' = \frac{\lambda_t}{c_p} \frac{\partial \tilde{H}}{\partial y} + (\mu_t - \frac{\mu_t}{Pr_t}) \left\{ \frac{\partial(\tilde{u}^2/2)}{\partial y} + \frac{\partial k}{\partial y} \right\} \quad (12)$$

where Pr_t is a turbulent Prandtl number relating the turbulent conductivity λ_t to the turbulent viscosity μ_t . The turbulent viscosity μ_t is assumed to be proportional to a turbulence velocity scale and a turbulence length scale. Following a proposal of (Jones, Launder 1972) the turbulence viscosity may be obtained from

$$\mu_t = C_{\mu} f_{\mu} \bar{\rho} \frac{k^2}{\varepsilon} \quad (13)$$

where ε is the turbulent dissipation rate. C_{μ} is a constant and f_{μ} is a function introduced to account for viscous effects in the near wall region.

THE k, ε MODEL

The transport equations for k and ε originally derived from the momentum equations are given in modelled form (Jones, Launder 1972), (Lam, Bremhorst 1981). Though the ε -equation can only be derived with the assumption $\nu_t = \mu_t/\rho = \text{const}$, it has been widely used for variable property flows (Hien, et.al., 1980), (Vandromme, Minh 1984), (Rodi, Scheurer 1985).

Turbulent energy equation

$$\text{div}(\bar{\rho}\tilde{\mathbf{w}}k) = \text{div}((\mu + \mu_t)\text{grad}k) + P_k - \bar{\rho}\varepsilon \quad (14)$$

$$P_k = -\bar{\rho}\tilde{\mathbf{w}}''\mathbf{w}'' : \text{Grad}\tilde{\mathbf{w}} \quad (15)$$

$$\bar{\rho}\varepsilon = \overline{\boldsymbol{\tau}'' : \text{Grad}\mathbf{w}''} \quad (16)$$

Turbulent dissipation rate equation

$$\text{div}(\bar{\rho}\tilde{\mathbf{w}}\varepsilon) = \text{div}((\mu + \frac{\mu_t}{\sigma_{\varepsilon}})\text{grad}\varepsilon) + C_1 f_1 \frac{\varepsilon}{k} P_k - C_2 f_2 \bar{\rho} \frac{\varepsilon^2}{k} + P_{\varepsilon} \quad (17)$$

$$P_{\varepsilon} = -2\boldsymbol{\tau}'' : \{ \text{Grad}\tilde{\mathbf{w}} \cdot \text{Grad}\mathbf{w}'' + \text{Grad}\mathbf{w}'' \cdot \text{Grad}\tilde{\mathbf{w}} \} \quad (18)$$

$$-2\boldsymbol{\tau}'' : \{ \mathbf{w}'' \cdot \text{Grad}(\text{Grad}\tilde{\mathbf{w}}) \} \quad (19)$$

The generation terms P_k and P_{ε} are written in their original form firstly to give attention to effects of acceleration. For the production of turbulent energy in a boundary layer the use of streamline quasiorthogonal coordinates leads to

$$P_k = - \widetilde{\bar{\rho}} u'' v'' \frac{\partial \tilde{u}}{\partial y} - \widetilde{\rho} u'' u'' \frac{\partial u}{\partial x} - \widetilde{\bar{\rho}} v'' v'' \tilde{u} \Gamma_{21}^2 \quad (20)$$

where Γ_{21}^2 is one of the Christoffel symbols arising from tensor analysis and might be interpreted as a measure of stream tube growth $\Gamma_{21}^2 \approx \frac{1}{\Delta y} \frac{\partial \Delta y}{\partial x}$ (Γ_{11}^1 is neglected $\Gamma_{11}^1 \approx 0$).

Elimination of Γ_{21}^2 using the equation of continuity yields

$$P_k = - \widetilde{\bar{\rho}} u'' v'' \frac{\partial \tilde{u}}{\partial y} - (\widetilde{\bar{\rho}} u'' u'' - \widetilde{\bar{\rho}} v'' v'') \frac{\partial \tilde{u}}{\partial x} + \widetilde{v'' v''} \tilde{u} \frac{\partial \bar{\rho}}{\partial x} \quad (21)$$

which is consistent with (Hanjalic, Launder 1980), (Rodi, Scheurer 1983), (Back, et.al., 1964) except for the variable density term being important in the supersonic region. The first term of P_k is the most important one in a boundary layer and is obtained without additional modelling assumptions

$$- \widetilde{\bar{\rho}} u'' v'' \frac{\partial \tilde{u}}{\partial y} = \mu_t \left(\frac{\partial \tilde{u}}{\partial y} \right)^2 \quad (22)$$

Concerning the remaining terms of P_k it is better to retain the view of P_k being an exchange term between the energy of turbulent motion and mean flow appearing with reverse sign in the equation of mechanical energy of the mean flow. In the case of a boundary layer ($u'' u'' > v'' v''$) in an accelerated flow ($\partial \tilde{u} / \partial x > 0$, $\partial \bar{\rho} / \partial x < 0$) the remaining terms of P_k are always negative transferring turbulent kinetic energy to mechanical energy of the mean flow.

The acceleration terms are of an order of magnitude smaller than the production P_k but the small imbalance $P_k - \bar{\rho} \varepsilon$ is influenced effectively.

However it would be hasty to rush to the conclusion that there is a decay of turbulence viscosity in an accelerated flow leaving the turbulent dissipation rate unconsidered.

There is a quite similar behaviour of the term P_ε in the dissipation rate equation and the idea for modelling is already given in (Hanjalic, Launder 1972). If the increase of the standard production of ε in the low Reynolds number region (Kebede et.al 1985) is assumed to be already considered when introducing the f_1 -function, and

$$- 2 \tau'' : \{ \mathbf{w}'' \cdot \text{Grad}(\text{Grad} \tilde{\mathbf{w}}) \} \quad (23)$$

is neglected, as in the standard model given by (Lam, Bremhorst 1981), P_ε reduces to the transfer term

$$P_\varepsilon = - 2 \tau'' : \begin{pmatrix} 2 \frac{\partial u''}{\partial x} \frac{\partial \tilde{u}}{\partial x} & \frac{\partial v''}{\partial x} \text{div} \tilde{\mathbf{w}} & \frac{\partial w''}{\partial x} \frac{\partial \tilde{u}}{\partial x} \\ \frac{\partial u''}{\partial y} \text{div} \tilde{\mathbf{w}} & 2 \frac{\partial v''}{\partial y} \tilde{u} \Gamma_{21}^2 & \frac{\partial w''}{\partial y} \tilde{u} \Gamma_{21}^2 \\ \frac{\partial u''}{\partial z} \frac{\partial \tilde{u}}{\partial x} & \frac{\partial v''}{\partial z} \tilde{u} \Gamma_{21}^2 & 0 \end{pmatrix} \quad (24)$$

with

$$\text{div} \tilde{\mathbf{w}} = \frac{\partial \tilde{u}}{\partial x} + \tilde{u} \Gamma_{21}^2 \quad (25)$$

Since the transfer term contains components of

$$\bar{\rho} \varepsilon = \tau'' : \text{Grad} \mathbf{w}'' = \tau'' : \begin{pmatrix} \frac{\partial u''}{\partial x} & \frac{\partial v''}{\partial x} & \frac{\partial w''}{\partial x} \\ \frac{\partial u''}{\partial y} & \frac{\partial v''}{\partial y} & \frac{\partial w''}{\partial y} \\ \frac{\partial u''}{\partial z} & \frac{\partial v''}{\partial z} & \frac{\partial w''}{\partial z} \end{pmatrix} \quad (26)$$

it seems reasonable to model the transfer term proportional to the dissipation rate and the acceleration of the flow.

In the present calculations, we used the compressible analogy to the relation (Hanjalic, Launder 1980), (Rodi, Scheurer 1983)

$$P_\varepsilon = - C_3 f_3 \frac{\varepsilon}{k} \left\{ \widetilde{\bar{\rho}} (u'' u'' - v'' v'') \frac{\partial \tilde{u}}{\partial x} - \widetilde{v'' v''} \tilde{u} \frac{\partial \bar{\rho}}{\partial x} \right\} \quad (27)$$

though the dependency on $u'' u''$, $v'' v''$ and on $\tilde{u} (\partial \bar{\rho} / \partial x)$ might be a subject for further discussion.

The constants and wall functions in the turbulence model equations (13), (14), (17), (21), (27) are taken from (Lam, Bremhorst 1981).

$$C_\mu = 0.09 \quad f_\mu = (1 - e^{-0.0165 R_k})^2 \left(1 + \frac{20.5}{R_t} \right) \quad (28)$$

$$\sigma_\varepsilon = 1.3 \quad (29)$$

$$C_1 = 1.44 \quad f_1 = 1 + \left(\frac{0.05}{f_\mu} \right)^3 \quad (30)$$

$$C_2 = 1.92 \quad f_2 = 1 - e^{-R_t^2} \quad (31)$$

$$R_t = \frac{k^2 \bar{\rho}}{\varepsilon \mu} \quad R_k = \frac{\sqrt{k} \gamma \bar{\rho}}{\mu} \quad (32)$$

Very little is known about the turbulence quantities $u'' u''$ and $v'' v''$ in accelerated flows appearing in the transfer terms in the turbulent transport equations. In accordance with measurements in zero pressure gradient flow, a turbulence structure coefficient for $u'' u''$ was used with a fixed value

$$\frac{\widetilde{u'' u''}}{k} = 1 \quad (33)$$

and $v'' v''$ was calculated from a standard relation e.g. (Ljuboja, Rodi 1981), (Gibson 1978) with the additional assumption $P_k \approx \bar{\rho} \varepsilon$

$$\frac{\widetilde{v'' v''}}{k} = 0.52 \left(\frac{1 - 0.25 f}{1 + 0.667 f} \right) \quad (34)$$

For the additional constant in the dissipation rate equation we used

$$C_3 = 4.44 \quad f_3 = 1 \quad (35)$$

as originally given by (Hanjalic, Launder 1980).

For the turbulent Prandtl number the equation

$$Pr_t = 0.67 \frac{1 + 0.675 f}{1 + 0.5 f} (1 + 0.167 f) \quad (36)$$

is used as developed by (Ljuboja, Rodi 1981) from an analysis of the $\overline{v''T''}$ equation. In a boundary layer

$$f = \frac{R_t}{3.72R_k} = \frac{k^{3/2}}{3.72\varepsilon y} \approx \frac{(-u''v'')^{3/2}}{0.41\varepsilon y} = \frac{L}{0.41y} = 1 \quad (37)$$

yields $Pr_t = 0.86$ as used in most calculations. In the outer region $f \rightarrow 0$ leads to $Pr_t = 0.67$ which is consistent with some measurements (Townsend 1976). The equation for Pr_t was used without modifications in the present calculations although other measurements (Fiedler 1974), (Chambers et.al. 1985) indicate that the turbulent Prandtl number in a turbulent mixing layer should be even lower $Pr_t \approx 0.5$ calling for a change of constants in the Pr_t formula.

INITIAL AND BOUNDARY CONDITIONS

Boundary conditions

The wall pressures obtained from the experiments were used as a boundary condition for the calculations performed with the parabolic finite difference code. In absence of curvature effects a cross-stream zero pressure gradient was assumed in the present calculations. The assumption of negligible curvature is also valid in the case of film cooling for velocity ratios less than 3 (Abramovich 1963). The measured wall temperatures were used as the thermal boundary condition, and the calculated wall heat flux is compared with the measured heat flux.

Velocity and kinetic energy were set to zero at the wall, and the dissipation rate of turbulent kinetic energy at the wall was calculated from the reduced turbulent kinetic energy equation

$$\varepsilon_w = \frac{\mu}{\rho} \left(\frac{\partial^2 k}{\partial y^2} \right)_w \quad (38)$$

All variables were submitted to a vanishing cross-stream gradient as a free stream boundary condition. In absence of cross-stream diffusive transport the mean flow equations reproduce the isentropic relation, and the turbulence transport equations automatically reduce to the decay equations

$$u_\infty \frac{\partial k}{\partial x} = -\varepsilon \quad (39)$$

$$u_\infty \frac{\partial \varepsilon}{\partial x} = -C_2 \frac{\varepsilon^2}{k} \quad (40)$$

Initial Conditions

As initial velocity profiles, the profile for a zero pressure gradient boundary layer developed on the surface from the suction to the coolant entry (fig.2) was used for the hot gas, and a velocity profile for fully developed channel flow was used for the cooling film. The mass flux of the coolant and the free stream velocity were obtained from the experiment. Since the slot was not removed in the absence of coolant injection, we assumed a slightly smaller initial boundary layer thickness. In the near wall regions the velocity profiles were determined by the law of the viscous sublayer.

The temperature profiles were obtained from the generalized

Crocco relation.

$$T = T_w + (T_r - T_w) \frac{u}{u_\infty} - r \frac{u^2}{2 c_p} \quad (41)$$

$$r = 0.88 \quad (42)$$

In the fully turbulent region the turbulent kinetic energy of the hot gas boundary layer is obtained from

$$k = \frac{\tau}{\sqrt{c_\mu} \rho} \quad (43)$$

with an additional wall damping correction.

The turbulent mixing length is defined by the ramp function $l = \min(0.41y, \lambda \delta)$, with λ as defined in (Crawford, Kays 1976). The turbulent dissipation rate is calculated from (Norris 1975):

$$\varepsilon = C^{1/4} \frac{k^{3/2}}{\mu} \left(1 + \frac{1.9}{Re_l} \right) \quad (44)$$

A smooth distribution of all variables between fully turbulent region and free stream was obtained using the Wake-function

$$W\left(\frac{y}{\delta}\right) = 2\left(3\left(\frac{y}{\delta}\right)^2 - 2\left(\frac{y}{\delta}\right)\right)^3 \quad (45)$$

The free stream value of the turbulent kinetic energy was obtained from $k_\infty = 3/2 Tu_\infty^2 u_\infty^2$ with $Tu_\infty = 1.5\%$ from the experiments of (Bauer et.al. 1980) in the same test section. The free stream dissipation rate of turbulent energy was calculated from the decay of grid turbulence:

$$\varepsilon_\infty = \frac{u_\infty k_\infty}{c_2 - 1} \frac{1}{x_t} \quad x_t = 0.5 \text{ m} \quad (46)$$

The turbulent kinetic energy at the entry of the cooling film was fixed to 5 %.

SOLUTION PROCEDURE

The mean flow equations for momentum and total enthalpy and the transport equations for turbulent energy and turbulent dissipation rate were solved on a streamline grid with a new computer code developed by the authors.

The finite difference equations were obtained by integrating the differential equations over the grid defined control volume according to the Gaussian integration rule

$$\iiint_V \text{div } \mathbf{vector} \, dV = \iint_S \mathbf{vector} \cdot \mathbf{n} \, dS \quad (47)$$

V : control volume

S : surface of control volume

\mathbf{n} : unity vector normal to the surface

Thus conservation of convective and diffusive transport is guaranteed automatically. The usual ω -transformation (Patanekar, Spalding 1970), (Crawford, Kays 1976) is omitted for simplicity and more attention to the slowly varying region of the viscous sublayer.

The finite difference equations are solved fully implicit in the

streamwise direction without iteration and interpolation for the boundary conditions.

In order to obtain grid-independent solutions a nonequidistant grid with 100 nodes was established in the cross stream direction with a constant spreading rate of 1.1 at the starting location.

The first grid node away from the wall was at a distance of $y^+ = 0.1$ approximately constant during the whole calculation. A constant forward step $\Delta x = 10^{-3}$ m was chosen in the present calculations giving a nondimensional step size of $300 < \Delta x^+ < 1000$. The results remained unchanged for smaller step size. The calculation time per forward step was 0.025 sec on a CDC Cyber 875 and the total computing time was 20 sec per run.

The independence of the results from the calculation procedure has been examined in comparison with a modified version of the GENMIX computer code (Patankar, Spalding 1970).

RESULTS

Heat transfer and film cooling has been studied for four different main stream conditions. Mass flow, stagnation pressure, stagnation temperature, average entry velocity and isothermal wall temperature for the heat transfer experiments are given table 1.

The measured pressures along the flat side of the nozzle are shown in fig.4. The acceleration parameter K is of the order 10^{-6} and a typical value at the throat is $0.3 \cdot 10^{-6}$. The wall heat flux measurements are plotted in fig.5 in comparison with the results calculated with the finite difference code.

In the case of film cooling, heat transfer results are shown for two main stream conditions in figs.7, 9, 11 and 13 for different coolant mass flow rates. The wall heat flux is plotted again in comparison with the finite difference calculations and the measured wall temperatures used as thermal boundary condition for the calculations are given in figs.6, 8, 10 and 12.

Injection mass flow, mass flow ratio, mass flux ratio, momentum flux ratio, injection velocity ratio, average injection velocity, injection temperature and temperature ratio at the entry of the injected cold air are given in table 2, 3, 4 and 5.

table 1 : mainstream conditions

Symbol	O	Δ	\square	\diamond	
\dot{m}	1.971	2.221	2.457	2.695	kg/s
p_o	190329	217227	242624	268616	Pa
T_o	447.4	458.6	467.8	476.3	K
u	55.34	56.02	59.02	59.60	m/s
T_w	309.5	309.5	318.0	323.0	K

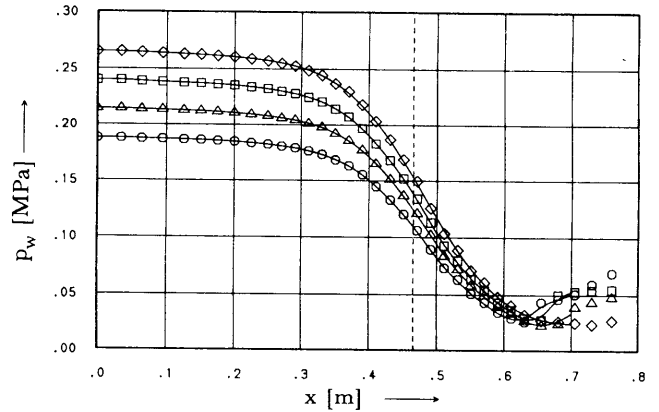


Fig.4 Wall pressure $p_w(x)$ along plane nozzle wall

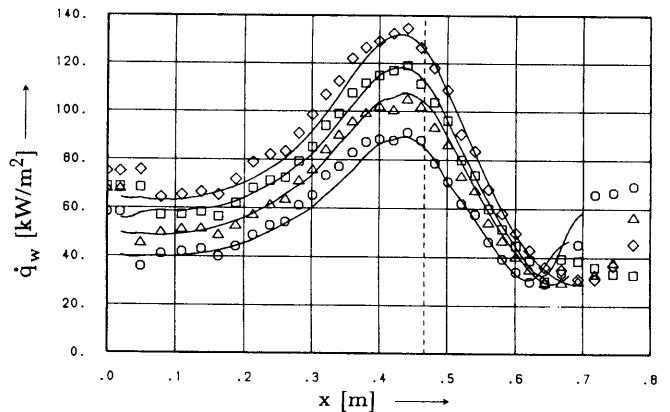


Fig.5 Wall heat flux $\dot{q}_w(x)$; $\diamond, \square, \Delta, O$ measurements; — finite difference calculation

table 2 : injection conditions for main stream mass flow $\dot{m} = 1.971 \text{ kg/s O}$; slot height $s = 5 \cdot 10^{-3} \text{ m}$

Symbol (figs.6,7)	X	Y	Z	
\dot{m}_c	0.084	0.059	0.045	kg/s
\dot{m}_c/\dot{m}	4.3	3.0	2.3	%
$\frac{(\rho u)_c}{\rho u}$	2.12	1.49	1.13	
$\frac{(\rho uu)_c}{\rho uu}$	2.88	1.45	0.84	
u_c/u	1.36	0.97	0.75	
u_c	75.3	53.8	41.3	m/s
T_c	293.3	296.6	298.1	kg/s
T_c/T	0.66	0.67	0.67	

table 3 : injection conditions for main stream mass flow $\dot{m} = 2.221 \text{ kg/s } \Delta$; slot height $s = 5 \cdot 10^{-3} \text{ m}$

Symbol (figs.8,9)	X	Y	Z	
\dot{m}_c	0.081	0.055	0.046	kg/s
\dot{m}_c/\dot{m}	3.7	2.5	2.1	%
$\frac{(\rho u)_c}{\rho u}$	1.79	1.22	1.01	
$\frac{(\rho uu)_c}{\rho uu}$	2.04	0.97	0.67	
u_c/u	1.14	0.79	0.66	
u_c	64.4	44.4	37.2	m/s
T_c	300.4	302.6	303.8	K
T_c/T	0.66	0.66	0.66	

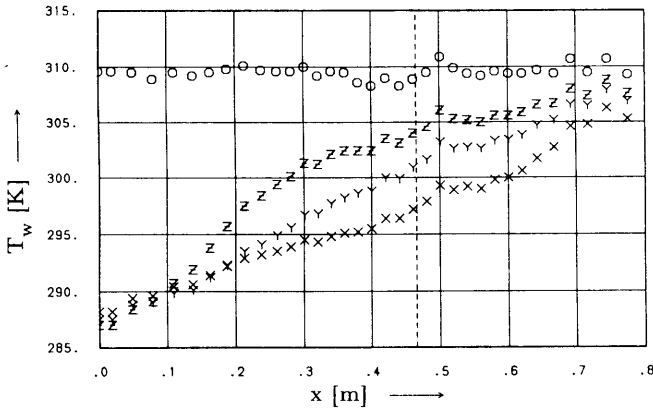


Fig.6 Wall temperature $T_w(x)$ along plane nozzle wall

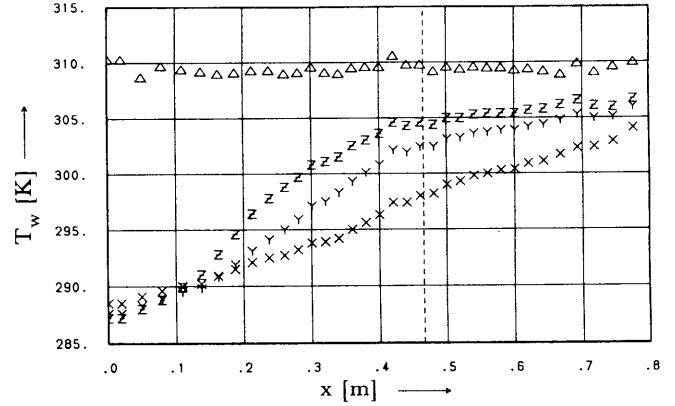


Fig.8 Wall temperature $T_w(x)$ along plane nozzle wall

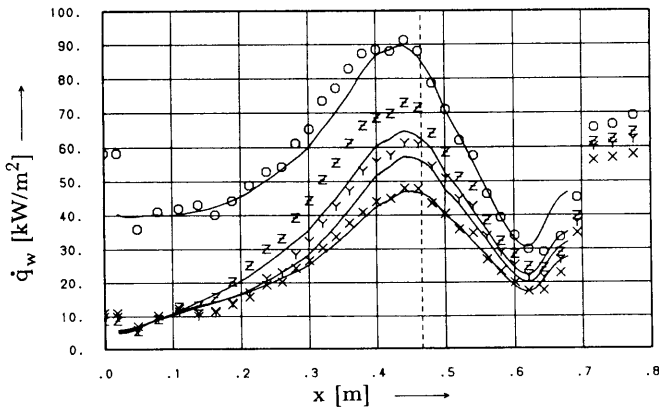


Fig.7 Wall heat flux $\dot{q}_w(x)$; O,X,Y,Z measurements; _____ finite difference calculation

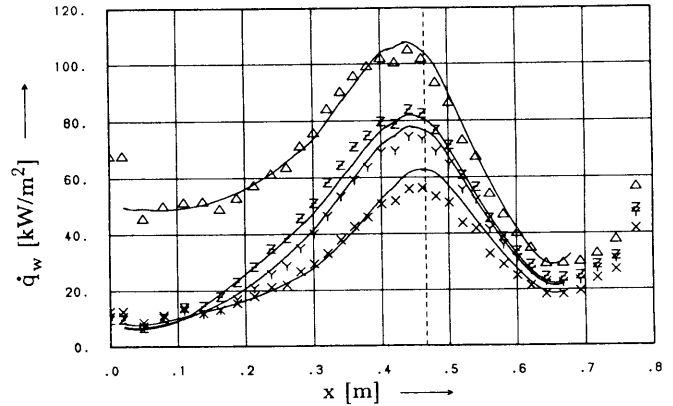


Fig.9 Wall heat flux $\dot{q}_w(x)$; Delta,X,Y,Z measurements; _____ finite difference calculation

table 4 : injection conditions for main stream mass flow $\dot{m} = 2.457 \text{ kg/s}$ \square ; slot height $s = 12 \cdot 10^{-3} \text{ m}$

Symbol (figs.10,11)	X	Y	Z	
\dot{m}_c	0.157	0.113	0.090	kg/s
\dot{m}_c/\dot{m}	6.4	4.6	3.6	%
$\frac{(\rho u)_c}{\rho u}$	1.25	0.90	0.71	
$\frac{(\rho uu)_c}{\rho uu}$	1.00	0.51	0.33	
u_c/u	0.79	0.57	0.46	
u_c	46.8	33.9	27.2	m/s
T_c	299.2	301.2	302.6	kg/s
T_c/T	0.64	0.65	0.65	

table 5 : injection conditions for main stream mass flow $\dot{m} = 2.695 \text{ kg/s}$ \diamond ; slot height $s = 12 \cdot 10^{-3} \text{ m}$

Symbol (figs.12,13)	X	Y	Z	
\dot{m}_c	0.149	0.115	0.087	kg/s
\dot{m}_c/\dot{m}	5.5	4.3	3.2	%
$\frac{(\rho u)_c}{\rho u}$	1.08	0.84	0.63	
$\frac{(\rho uu)_c}{\rho uu}$	0.73	0.44	0.25	
u_c/u	0.67	0.53	0.40	
u_c	40.2	31.3	23.7	m/s
T_c	299.5	300.9	302.4	kg/s
T_c/T	0.63	0.63	0.64	

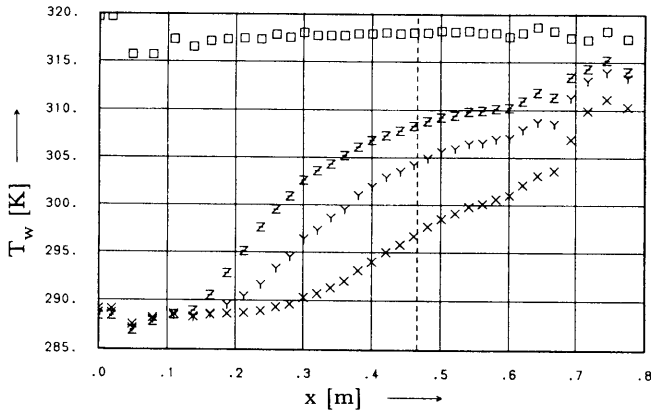


Fig.10 Wall temperature $T_w(x)$ along plane nozzle wall

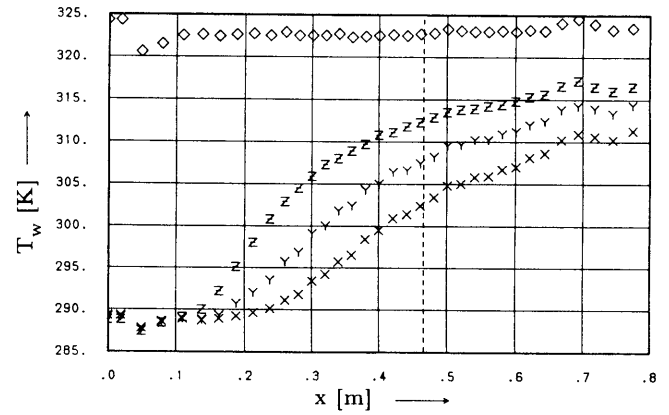


Fig.12 Wall temperature $T_w(x)$ along plane nozzle wall

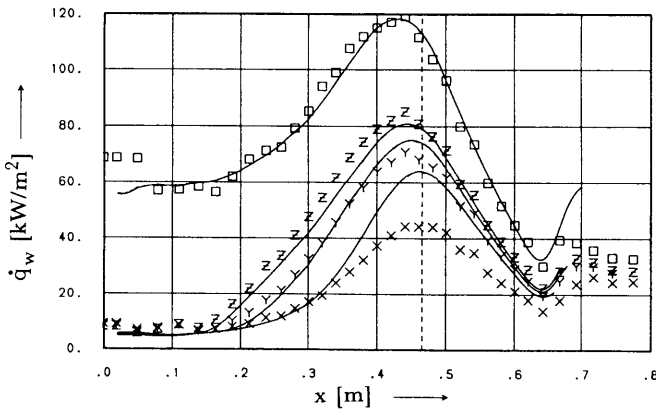


Fig.11 Wall heat flux $\dot{q}_w(x)$; \square, X, Y, Z measurements;
 — finite difference calculation

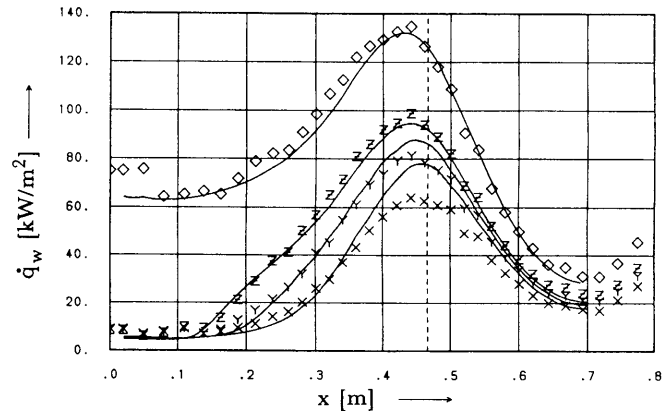


Fig.13 Wall heat flux $\dot{q}_w(x)$; \diamond, X, Y, Z measurements;
 — finite difference calculation

CONCLUSIONS

Experiments and calculations on heat transfer and film cooling stability in a convergent – divergent nozzle are presented.

Heat Transfer

The maximum wall heat flux is determined slightly upstream the throat as indicated in earlier measurements. In contrast to many simple correlation methods finite difference calculations predict the location of the maximum very well, but most standard turbulence models e.g. (Crawford, Kays 1976), (Norris 1975), (Norris, Reynolds 1975), (Jones, Launder 1972), (Lam, Bremhorst 1981), (Chien 1982) predict the wall heat flux approximately 10 % too small. The underprediction of heat transfer has also been observed by (Crawford 1986) comparing calculations with WR (Wilcox, Rubesin 1980), JL (Jones, Launder 1972) and CH (Chien 1982) two equation turbulence models with the favourable pressure gradient experiments of (Julien et.al. 1969) and (Thielbar et.al. 1969). (Kays et.al.1970) have used a zero – equation model very similar to the STAN5 zero – equation model (Crawford, Kays 1976), but with a higher Karman constant (0.44) in comparison to their measurements of the heat transfer to a highly accelerated turbulent boundary layer. This modification also improves the agreement between calculation and our measurements.

In the present investigation, we used a standard model (Lam, Bremhorst 1981), but we retained the acceleration terms appearing in the turbulence transport equations and we used $C_3 = 4.44$ as originally given by (Hanjalic, Launder 1980). If the turbulence structure is assumed to be similar to the structure of a fully turbulent zero pressure gradient boundary layer, the wall heat flux is predicted very well.

Film Cooling

In the case of film cooling there is no influence of the throat on the cooling film stability. It should be pointed out that an injection mass flow ratio of only $\dot{m}_c/\dot{m} = 2.1$ % (table 3) yielded a significant decrease in heat transfer as well in the subsonic as in the supersonic region. An approximate comparison of mass flow ratios for axisymmetric nozzles may be obtained with simple geometry relations.

The calculated results are fairly well but not yet fully satisfactory for all injection rates. In contrast to the standard models predicting the wall heat flux again too small, the model with the additional acceleration terms overpredicts the heat transfer in the throat region for high injection rates.

Turbulence Model

The calculations are very sensitive to the choice of the additional constant and C_3 should be finally fixed from comparison of predictions with experiments for many different accelerated flow cases. Assumed the turbulence structure is described well with equations (33) and (34), the overprediction of heat transfer for high injection rates indicates that the constant $C_3 = 4.44$ as originally given by (Hanjalic, Launder 1980) might be too big.

Since acceleration acts in a different way on $\widetilde{u''u''}$ and $\widetilde{v''v''}$ it might also be necessary to examine the structure of accelerated boundary layers and mixing layers experimentally and to use a complete Reynolds stress model. Additionally it

will be necessary to develop an $\widetilde{v''T''}$ – equation for accelerated flows and to prove the concept of a turbulent Prandtl number. Finally it is important to improve the standard models in comparison with experiments in zero pressure gradient large density ratio boundary layers.

REFERENCES

- Abramovich, G.N.(1963), "The Theory of Turbulent Jets", The M.I.T.Press, Massachusetts Institute of Technology, Cambridge, Massachusetts
- Back, L.H., Massier, P.F., Gier, H.L.(1964), "Convective Heat Transfer in a Convergent – Divergent Nozzle", Int. Journal Heat Mass Transfer Vol.7 pp.549 – 568
- Back, L.H., Cuffel, R.F., Massier, P.F.(1970), "Laminarization of a Turbulent Boundary Layer in Nozzle Flow – Boundary Layer and Heat Transfer Measurements With Wall Cooling", ASME: Journal of Heat Transfer, Vol.92, pp.333 – 344
- Back, L.H., Cuffel, R.F.(1972), "Turbulent boundary layer measurements along a supersonic nozzle with and without wall cooling", ASME: Society of Mechanical Engineering, Ser. C, 94, 242 – 243, (CAT 7207)
- Bauer, K., Winkler, W., Grigull, U.(1978), "Heat transfer in compressible turbulent boundary layer", Proceedings International Heat Transfer Conference, Toronto, Vol.5 pp.191 – 196
- Bauer, K., Straub, J., Grigull, U.(1980), "Influence of Free – Stream Turbulence Intensity on Heat Transfer in the Two – Dimensional Turbulent Boundary Layer of a Accelerated Compressible Flow", Int. Journal Heat Mass Transfer Vol.23 pp.1635 – 1642
- Boldman, D.R., Neumann, H.E., Schmidt, J.F.(1967), "Heat transfer in 30° and 60° half – angle convergence nozzles with various diameter uncooled pipe inlets", NASA TN D – 4177
- Boldman, D.R., Graham, R.W.(1972), "Heat transfer and boundary layers in conical nozzles", NASA TN D – 6594
- Chambers, A.J., Antonia, R.A., Fulachier, L.(1985), "Turbulent Prandtl number and spectral characteristics of a turbulent mixing layer", Int.Journal Heat Mass Transfer Vol.28, No 8, pp.1461 – 1468
- Chien, K. – Y.(1982), "Predictions of Channel and Boundary – Layer Flows with a Low – Reynolds – Number Turbulence Model", AIAA Journal, 20, p.33
- Crawford, M.E., Kays, W.M.(1976), "STAN5 – A Program for Numerical Computation of Two – Dimensional Internal and External Boundary Layer Flows", NASA Contractor Report CR – 2742
- Crawford, M.E.(1986), "Lecture Series 1986 – 06: Convective Heat Transfer and Film Cooling in Turbomachinery", von Karman Institute for Fluid Dynamics, Brussels – Belgium
- Favre, A.(1965), "Equations des gaz turbulent compressible", part I: Journal de Mecanique: Vol.4, Nr.3 pp.361 – 390 part II: Journal de Mecanique: Vol. 4, Nr.4 pp 391 – 421

- Favre A.(1975), "Equations statistiques des fluides turbulents compressibles", Veme Congres canadien de Mecanique Appliquee, New Brunswick, Proc. pp. G3 = G34
- Fiedler, H.E.(1974), "Transport of heat across a plane turbulent mixing layer", Adv. Geophys 18,93 – 109
- Gibson, M.M.(1978), "An Algebraic Stress and Heat Flux Model for Turbulent Shear Flow with Streamline Curvature", Int.Heat Mass Transfer, Vol.21, pp.1609 – 1617
- Hanjalic, K., Launder, B.E.(1980), "Sensitizing the Dissipation Equation to Irrotational Strains", ASME: Journal of Fluids Engineering Vol.102 pp.34 – 40
- Hay, N., Lampard, D., Saluja, C.L.(1985), "Effects of the condition of the Approach Boundary Layer and of Mainstream Pressure Gradients on the Heat Transfer Coefficient on Film – Cooled Surfaces", ASME: Journal of Engineering for Gas Turbines and Power Vol.107 pp.99 – 110
- Hien, H.M., Vandromme, D.D., MaxCormack, R.W.(1980), "Comparison of computation with experiment", ASFOSR – IFP Conference proceedings, Stanford
- Jones, T.V., Forth, C.J.P.(1986), "The Scaling of Film Cooling – Theoretical and Experimental Results", Lecture Series 1986 – 06: Convective Heat Transfer and Film Cooling in Turbomachinery, von Karman Institute for Fluid Dynamics, Brussels – Belgium
- Jones, W.P., Launder, B.E.(1972), "The prediction of laminarization with a two equation model of turbulence", International Journal of Heat and Mass Transfer Vol.15 pp.301 – 314
- Julien, H.L., Kays, W.M. Moffat, R.J.(1969), "The Turbulent Boundary Layer on a Porous Plate: Experimental Study of The Effects of a Favourable Pressure Gradient", Thermosciences Division, Department of Mechanical Engineering, Stanford University, Report Nr.HMT – 4
- Kays, W.M., Moffat, R.J., Thielbar, W.H. (1970), "Heat Transfer to the Highly Accelerated Turbulent Boundary Layer With and Without Mass Addition, ASME: Journal of Heat Transfer, Vol.92, pp.499 – 505
- Kebede, W., Launder, B.E., Younis, B.A.(1985), "Large – Amplitude Periodic Pipe Flow: A Second – Moment Closure Study", Proc.5th Turbulent Shear Flow Symposium, pp.16.23 – 16.29, Cornell
- Klingbeil, E.(1966), "Tensorrechnung für Ingenieure", Bibliographisches Institut AG, Mannheim
- Lam, C.K.G., Bremhorst, K.(1981), "A Modified Form of the $k - \epsilon$ Model for Predicting Wall Turbulence", ASME: Journal of Fluids Engineering Vol.103 pp.456 – 460
- Ligrani, P.M., Camci, C.(1985), "Adiabatic Film Cooling Effectiveness From Heat Transfer, Measurements in Compressible Variable – Property Flow", ASME: Journal of Heat Transfer Vol.107 pp.313 – 320
- Ljuboja, M., Rodi, W.(1980), "Calculation of Turbulent Wall Jets With an Algebraic Reynolds Stress Model", ASME: Journal of Fluids Engineering Vol. 102 pp.351 – 356
- Ljuboja, M., Rodi, W.(1981), "Prediction of Horizontal and Vertical Turbulent Buoyant Wall Jets", ASME: Journal of Heat Transfer Vol.103, pp.343 – 349
- Norris, L.(1975), "Turbulent Channel Flow with a Moving Wavy Boundary", Dissertation, Department for Mechanical Engineering, Stanford University, Stanford
- Norris, L.H., Reynolds, W.C. (1975), "Turbulent Channel Flow with a Moving Wavy Boundary", Rept. – No. FM – 10, Standford University, Dept.Mech.Eng., Standford, CA.
- Patankar, S.V., Spalding, D.B.(1970), "Heat and Mass Transfer in Boundary Layers – A General Calculation Procedure", Intertext Books, London
- Patel, V.C.; Rodi, W., Scheurer, G.(1985), "Turbulence Models for Near – Wall and Low Reynolds Number Flows: A Review", AIAA Journal Vol.23 No.9 pp.1308 – 1319
- Rodi, W., Scheurer, G.(1983), "Scrutinizing the $k - \epsilon$ – Model under Adverse Pressure Gradient Conditions", Proc.4th Turbulent Shear Flow Symposium, Karlsruhe
- Rodi, W., Scheurer, G.(1985), "Calculation of Heat Transfer to Convection – Cooled Gas Turbine Blades", ASME: Journal of Engineering for Gas Turbines and Power Vol.107 pp.620 – 627
- Rotta, J.C.(1972), "Turbulente Strömungen – Eine Einführung in die Theorie und ihre Anwendungen", B.G.Teubner, Stuttgart
- Rued, K., Wittig, G.(1985), "Free – Stream Turbulence and Pressure Gradient Effects on Heat Transfer and Boundary Layer Development on Highly Cooled Surfaces", ASME: Journal of Engineering for Gas Turbines and Power Vol.107 pp.54 – 59
- Thielbar, W.H., Kays, W.M., Moffat, R.J.(1969), "The Turbulent Boundary Layer: Experimental Heat Transfer With Boiling, Suction, and Favourable Pressure Gradient", Thermosciences Division, Department of Mechanical Engineering, Stanford University, Report Nr.HMT – 5
- Townsend, A.A.(1976), "The Structure of Turbulent Shear Flow", Cambridge University Press, Great Britain, ISBN 0521 20710 X ISBN 0521 29819 9
- Vandromme, D.D., Minh, H.H.(1984), "Solution of the Compressible Navier – Stokes Equations: Applications to Complex Turbulent Flows", Lecture Series 1984 – 04: Computational Fluid Dynamics, von Karman Institute for Fluid Dynamics, Brussels – Belgium
- Wang, T., Simon, T.W., Buddhavarapu(1985), "Heat Transfer and Fluid Mechanics Measurements in Transitional Boundary Layer Flows", ASME: Journal of Engineering for Power Vol.104 pp.743 – 750
- Wilcox, D.C., Rubesin, M.W.(1980), "Progress in Turbulence Modeling for Complex Flow Fields Including Effects of Compressibility", NASA, TP 1517
- Winkler, W., Grigull, U.(1977), "Wärmeübergang turbulenter kompressibler Grenzschichtströmungen mit starken negativen Druckgradienten (Wärmeübergang in einer Laval-düse)", Wärme – und Stoffübertragung 10, pp.281 – 291