Heat Transfer during Founding and Cooling in Glass Processing

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Abstract

Process analysis and numerical simulation are applied to the heat transfer in a particular production process of lead crystal glass to improve product quality on the one hand and to reduce the consumption of energy on the other hand. A theoretical analysis supported by temperature measurements along the production line demostrates that the control of the mold and the stamper surface-temperature strongly affects the quality of the product.
The major heat transfer phenomenon involved is transient heat conduction, accompanied by the boundary conditions of free, forced and mixed convection and radiation. Moreover, the time-dependent boundary conditions and the heating and cooling during the production process are also taken into consideration. Furthermore this paper points out fundamental considerations in grid generation to model the time-dependent boundary conditions. The finite-volume program is designed to run both on large scale computers and, with some limitations in the total amount of grid points, on personal computers.
Revealing the fundamental heat transfer mechanism of this process, criteria for decision in the analysis of the whole production system are given. Detailed information about the development of the computer program, its advantages and qualities are provided. Examples to improve the product quality by using the simulation program are given, and finally the good correspondence between calculation and measurement will be pointed out.

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List of Symbols

A	area
a	coefficient
c	specific heat capacity
$f_{r,P}$	interpolation factor
H	height
Nu	Nusselt-number
Pr	Prandtl-number
Ra	Rayleigh-number
R	radial coordinate
T	temperature
t	time
V	volume
Z	axial coordinate
α	heat transfer coefficient
Δ	increment
ϵ	emission coefficient
λ	thermal conductivity
η	dynamic viscosity
ϱ	density
σ_S	Stefan Boltzman factor
Φ	arbitrary variable

List of Subscripts

con	convective heat transfer
E	east
N	north
new	actual time step
old	previous time step
rad	radiative heat transfer
S	south
$wall$	wall
W	west
∞	surrounding

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1. Introduction

The results presented in this paper are part of a research project of the Bavarian Research Cooperative of Systems Engineering. The fundamental aim of the project is the optimization of energy demand, the minimization of exhaust and simultaneous improving of product quality.

include figure 1

As shown in figure 1 an analysis of the whole production line makes the energy flow and the energy demand very transparent. The figure shows that a large amount of the energy losses are surface losses caused by radiation and convection, which can hardly be avoided. Most of the energy loss caused by the exhaust gas is already recycled by the heat exchanger used for the preheating of the combustion air. On the other hand the reduction of the loss caused by inferior quality can save about seven percent of the energy input. Product quality means an excellent smooth product surface without any ripples or pitting marks. Improving the product quality is also an important economic factor. Experience shows that the pressing process has an important influence on the product quality, because this production step is responsible for the resulting product surface.

Detailed measurements of the temperature distribution of the mold and a simultaneous observation of the product quality pointed out that the temperature control of the mold and stamper surface strongly affects the quality of the raw glass body surface. Quality problems are caused by two different effects. When the mold temperature is too low the surface of the glass drop solidifies before completely covering the mold surface. The result is an uneven glass body surface of inferior quality (Fig 2).

include figure 2

The second effect is a surface temperature above the optimal temperature of the press mold. If the mold temperature is too high, the raw glass body sticks to the mold and can hardly be removed. Especially an adjustment of the surface temperatures of the mold and the stamper seems to be very important for the product quality. A temperature adjustment can be achieved by optimizing the process times and by optimizing the outer surface of the mold by means of design. For this modification a detailed knowledge of the temperature distribution in the mold is necessary. Therefore a computer code was developed to calculate the temperature distribution within the mold. This program allows the calculation of molds of arbitrary shape and is thus an important tool for the designer when layouting the outer shape of the mold. This is the main reason why the computer program has to run on an personal computer.

2. Physical Model

2.1 Governing conservation equations

The aim of the numerical simulation is the calculation of the temperature distribution in glass mold and stamper during the process. Therefore the differential equation of the diffusive heat transport has to be solved. According to [1] the convective energy transport can be neglected after the solidification.

Thus the heat transfer in the glass can be described by Fourier's law of heat conduction. Because of the rotational symmetry of the mold the two dimensional heat diffusion equation in cylindrical coordinates is employed:

$$c \cdot \varrho \cdot \frac{\partial T}{\partial t} = \frac{1}{R} \cdot \frac{\partial}{\partial R} \cdot \left(\lambda \cdot R \cdot \frac{\partial T}{\partial R}\right) + \frac{\partial}{\partial Z} \cdot \left(\lambda \cdot \frac{\partial T}{\partial Z}\right) \qquad (1)$$

2.2 Boundary and initial conditions

The calculation domain is shown in figure 3. The calculation domain is chosen to be rectangular, control volumes, that cover the air-regions of the calculation domain are blocked off, set to a constant air temperature so that, while numerical calculations are performed these regions are not taken into calculation, helping reduce computing time. The production process is divided into several steps with different boundary conditions.

include figure 3

In the first production step (Fig. 3 a.) the mold is filled with liquid glass. The initial temperature distribution for the glass, the steel mold and the surrounding air is given. In the second step the metal stamper with an estimated initial temperature distribution forms the raw glass body (Fig. 3 b.). An estimation of the Peclet-number ($5.6 \cdot 10^3$) and of the temperature profile in the glass shows, that the flow of the glass during the process in which the metal stamper is pressing the glass from the bottom of the mold up along the side walls can be treated as plug flow. This also is confirmed by an estimation of the penetration depth of the temperature during the short duration of this step ($< 1 sec.$) (Fig. 4). Therefore for the calculation of the heat transfer within the glass plug and between the glass plug and the mold only heat conduction is regarded. In addition, because of the very short duration of this step , the error of the approximation has nearly no influence on the final results of the whole process simulation. The temperature of the displaced glass is set to the average temperature of the elements in the glass not occupied by the stamper. In the third step (Fig. 3 c) the stamper is removed and the raw glass body is cooled by free convection and radiation. In the fourth step (Fig. 3 d) the raw glass body also is removed and the mold is cooled by the surrounding air.

include figure 4

After the fourth step the process is started again. The initial temperatures for the next cycle are those calculated in the cycle before. Therefore the temperature distribution in the stamper is calculated simultaneously to the distribution in the mold during steps three and four.

The left boundary west side of the mold is treated as a symmetry boundary, that means the heat flux in normal direction to the west side is set to zero (von Neumann condition).

$$\left(\frac{\partial T}{\partial R}\right)_{R=0} = O \tag{2}$$

The lower boundary south side of the mold is treated as an outlet boundary with a user defined heat flux in normal direction, which can be set for each element individually.

Heat transport by free convection and radiation occurs along the boundaries between steel and air, and during step one and three between glass and air respectively. The resulting heat transfer coefficient is the sum of the convective α_{con} and radiative heat transfer coefficients α_{rad}.

$$\alpha = \alpha_{rad} + \alpha_{con} \tag{3}$$

The convective heat transfer coefficient due to free convection α_{con} depends on the temperature of the surroundings and the wall and the thermo-physical properties. The dependance of the thermo-physical properties on temperature and glass composition is considered too. The density is calculated using a model introduced by Gilard P. and Dubrul L. [5], the coefficient of extension is calculated according to Debye and Zwicker [5], the specific heat according to Schwiete and Ziegler [5], the viscosty according to

Šašec [5] and the heat conductivity according to E.H. Ratcliffe [5].
For the calculation of this heat transfer coefficient several different modells were compared. For example free convection at a vertical cylinder (Churchill + Chu) [2] or free convection at a vertical plate (Churchill + Chu) [2]. Best results were reached by using the model of free convection for a sphere introduced by Krischer [2], with:

$$Nu = \left\{ 1.414 + \frac{0.387 \cdot Ra^{\frac{1}{6}}}{\left(1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right)^{\frac{8}{27}}} \right\}^2 \tag{4}$$

$$\alpha_{con} = \frac{Nu \cdot \lambda}{H} \tag{5}$$

For modelling the heat transfer by radiation a heat transfer coefficient α_{rad} is introduced:

$$\alpha_{rad} = \sigma_{12} \cdot (T_{wall} + T_\infty) \cdot \left(T_{wall}^2 + T_\infty^2\right) \tag{6}$$

For the calculation of the radiation factor σ_{12} two different models were taken into consideration. Radiation between two parallel surfaces with

$$\sigma_{12,I} = \frac{\epsilon_1 \cdot \sigma_S}{2 - \epsilon_1} \tag{7}$$

and radiation between two surfaces with area $A_1 \ll$ area A_2

$$\sigma_{12,II} = \epsilon_1 \cdot \sigma_S \tag{8}$$

with the Stefan-Boltzmann factor

$$\sigma_S = 5.67 \cdot 10^{-8} W/m^2K^4 \tag{9}$$

A comparison between these models shows that the radiation factors vary only in a range of about five percent depending on the calculation model.

$$\sigma_{12,I} = 0,96 \cdot \sigma_{12,II} \tag{10}$$

Because the resulting heat transfer coefficient eq.(3) depends on the unknown wall temperature, it is calculated iteratively, by adjusting α so that the heat flux due to conduction normal to the mold-wall is equal to the heat flux due to convection for the boundary elements.

2.3 Numerical solution

Equation (1) is numerically solved by a flux conservative finite volume method [3]. In a first step the calculation domain is divided up into rectangular control volumes, with the numerical gridpoints in the center of each volume. In a second step the transport equation (1) has to be discretizated by integrating every part of the equation over the control volume. By employing Gauss' integral theorem an integro differential equation (11) is obtained, where the temperature change with time is independent from the diffusive heat flux through the borders of the control volumes.

$$\int_{\partial V} c \cdot \varrho \cdot \frac{\partial T}{\partial t} dV =$$

$$\int_S^N \left(\left(\lambda \cdot \frac{\partial T}{\partial R} \right)_E - \left(\lambda \cdot \frac{\partial T}{\partial R} \right)_W \right) \cdot R \cdot dZ + \int_W^E \left(\left(\lambda \cdot \frac{\partial T}{\partial Z} \right)_N - \left(\lambda \cdot \frac{\partial T}{\partial Z} \right)_S \right) R \cdot dR \tag{11}$$

Finally this integro differential equation is discretizated instead of the original partial differential equation. The method is conservative, because each heat flux which leaves one control volume flows into the neighbouring volume; this holds for the whole calculation domain. The discretized equation for a central grid point P is given by:

$$a_P \cdot T_P^{new} = a_W \cdot T_W^{new} + a_E \cdot T_E^{new} + a_S \cdot T_S^{new} + a_N \cdot T_N^{new} + b_P \tag{12}$$

with the finite-volume coefficients:

$$a_N = R_P \cdot \Delta R \cdot \frac{\lambda_n}{\varrho_n \cdot c_n \cdot \Delta Z_n} \tag{13}$$

$$a_S = R_P \cdot \Delta R \cdot \frac{\lambda_s}{\varrho_s \cdot c_s \cdot \Delta Z_s} \tag{14}$$

$$a_E = R_i \cdot \Delta Z \cdot \frac{\lambda_e}{\varrho_e \cdot c_e \cdot \Delta R_e} \tag{15}$$

$$a_W = R_{i-1} \cdot \Delta Z \cdot \frac{\lambda_w}{\varrho_w \cdot c_w \cdot \Delta R_w} \tag{16}$$

$$b_P = a_P^{old} \cdot T_P^{old} \tag{17}$$

$$a_P = a_N + a_S + a_E + a_W + a_P^{old} \tag{18}$$

$$a_P^{old} = \frac{R \cdot \Delta R \cdot \Delta Z}{\Delta t} = \frac{\Delta V}{\Delta t} \tag{19}$$

The subscript "*old*" marks the previous time step, the subscript "*new*" marks the actual time step. For each grid point a seperate equation can be derived resulting in a system of linear equation with the grid point temperatures as the solution's vector. A pentadiagonal coefficient matrix is thus obtained.

This sytem of equations can be solved by several algorithms. Here the Stone [4] algorithm was used, which decomposes the coefficient matrix into a lower and upper triangular matrix.

As mentioned before, the diffusive heat flux is calculated at the border of the control volumes. Therefore the knowledge of the temperatures and the thermo physical properties at the borders is necessary. However only the values at the center grid points are known. The corresponding values at the borders are calculated by an interpolation method, first introduced by [3]; this is demonstrated for the east border (Φ represents an arbitrary variable for example temperature):

$$\Phi = \frac{1}{\frac{(1-f_{r,P})}{\Phi_P} + \frac{f_{r,P}}{\Phi_E}} \tag{20}$$

with the interpolation factor $f_{r,P}$

$$f_{r,P} = \frac{R_i - R_{i-1}}{R_{i+1} - R_{i-1}} \tag{21}$$

With this interpolation method even large differences in thermo physical properties between two neighbouring control volumes can be treated, for example at the border between steel and glass, without loosing numerical stability [3].

By observing the solidification of the glass which is defined as a temperature of 574°C (depending on the composition, $\eta = 10^{13}\ dPas$) at which glass can be regarded as technically solid the time when the calculated results reach a steady state can be obtained.

include figure 5

Figure 5 shows the solidification of the glass for several calculated production cycles. It is shown, that there is nearly no further alteration of the solidification rate after the third calculated production cycle. This is used as an abortion criterion.

3. Results

The calculated test mold is approximated by a grid of 76 control volumes in vertical and 65 in horizontal direction. This leads to control volumina of 2 mm in width and height . The initial temperature of the liquid glass is 1100 °C and the initial temperature of the mold is set to 350 °C for the first production cycle.

On the left side of figure 6 the calculated temperature distribution of a press mold as used in the production process is shown. The duration of one production cycle is 80 seconds. So the figure shows the 32nd second of the 6th production cycle when the stamper is already removed. In the figure the problem zones of the processing mold become obvious. In the transition from the horizontal to the vertical region of the mold, the temperture is too low and in the bottom region the temperature is too high. This causes quality problems as mentioned before. To reach an adjustment of the temperature distribution we made two optimization-suggestions.

First we changed the wall thickness of the mold to increase its energy storage. Additionally we simulated a cooling of the bottom of the press mold. As shown in the figure the temperatures of the bottom region could be decreased and the temperature in the transition from the horizontal to the vertical region could be increased by these measures.

include figure 6

This example proves that using this computer simulation helps to realize a temperature distribution, that is expected to improve the product quality.

4. Verification of the calculated results by measurements during production

In order to verify and adapt the code various measurements were performed during production. Thirty-six thermocouples were installed in a mold and with these thermocouples the temperature distribution during various production cycles were measured. Figure 7 shows the measured and calculated temperture profile for one point and proves the expected good correspondence between calculation and reality.

include figure 7

5. Conclusion

The presented computer program is an important tool for the designer of molds used for pressing of rotational symmetric glass bodies. The knowledge of the temperature distribution allows control of the surface temperatures by design measures.
With these optimized molds the product quality can be increased and the rate of waste can be decreased and thereby the energy demand can be improved as well.

The next step is to find out whether similar good results could also be obtained for molds of quite different shapes. This is very important because this computer program could not only be very useful for the production of glass for daily use but also, for example, for the production of dispersion glasses in the automobile industry, which exhibit very different diameter to height proportions. Therefore further experiments are necessary to prove the numerical results for other mold shapes for a more extensive use of the program.

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According to Schlünder [6] and because of the high Prandtl-number of the glass, the convective heat transfer in the glass while the metal stamper is pressing the glass from the bottom of the mold up along the side walls is treated as an insertion of a solid matter.