Chapter 14

Scaling and Modelling Laws in Two-Phase Flow and Boiling Heat Transfer

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14.1 Summary

In two-phase flow scaling is much more limited to very narrowly defined physical phenomena than in single phase fluids. For complex and combined phenomena it can be achieved not by using dimensionless numbers alone but in addition a detailed mathematical description of the physical problem – usually in the form of a computer program – must be available. An important role is played by the scaling of the thermodynamic data of the modelling fluid. From a literature survey and from own scaling experiments the conclusion can be drawn that Freon is a quite suitable modelling fluid for scaling steam – water mixtures. However, without a theoretical description of the phenomena nondimensional numbers for scaling two-phase flow must be handled very carefully.

14.2 Introduction

The transfer of experimental results obtained under scaled conditions to the reality in an industrial plant is an old problem of engineering. There must be a sufficient similarity of the main parameters influencing the precedents in the test equipment and in the original set-up. This similarity can be of geometrical, mechanical, static, dynamic, thermal, thermodynamic, electrical and mechanical nature.

In single phase hydrodynamics and heat transfer we are accustomed to use scaling and modelling laws since many years. The occurrences are analogous in a diabatic flow if

the velocity field,

the temperature field and

the pressure field (supersonic flow)

are similar. Only regarding subsonic flow the velocity and temperature fields are described by the well-known constitution laws for mass, energy and momentum. From these laws dimensionless numbers like

$$\frac{x}{l} \ , \ \frac{y}{l} \ , \ \frac{z}{l}$$

$$Re = \frac{w \cdot l}{\nu} \quad , \quad Fr = \frac{w^2}{gl} \quad , \quad Eu = \frac{\Delta p}{\rho w^2}$$

$$Nu = \frac{\alpha \cdot l}{\lambda} \quad , \quad Gr = \frac{l^3 g \beta \nu}{\nu^2} \quad , \quad Pr = \frac{\eta \cdot c}{\lambda}$$

are derived and similarity is assumed, if these dimensionless numbers are identical in the test and in the original conditions. Usually it is easy to guarantee the geometrical similarity by choosing the same ratios of length x/ , y/ , z/ , i.e. to make all dimensions of the test object proportional to a characteristical length l of the original set-up. The other dimensionless numbers represent ratios of forces or of energies (e.g. the Nusselt-number) which cannot be made identical in the model and in the original as it can easily be seen from the different exponents of the parameters. The velocity is in the Reynolds-number of the first power and in the Froude-number of the second one. The Reynolds-number is the ratio of inertia and viscous forces and the Froude-number contains the buoyancy force. This means, that in mixed convection either the forced flow or the buoyant induced one can be scaled only. In addition the velocity and the temperature field is influenced by the thermodynamic properties - in single phase flow mainly the viscosity, density, specific heat and thermal conductivity. By comparing the behaviour of different fluids not all these properties can be exactly scaled in general.

The velocity and the temperature field can be described by functions of the form

$$\frac{w}{w_o} = f_w \ (\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \ Re, \ Pr, \ Gr) \qquad (14.1)$$

$$\frac{\nu}{\nu_o} = f \ (\frac{x}{l}, \frac{y}{l}, \frac{z}{l}, \ Re, \ Pr, \ Gr) \qquad (14.2)$$

$$Nu = f_\alpha \ (Re, \ Pr, \ Gr) \qquad (14.3)$$

and in practice we rule out one of the dimensionless numbers, i.e. the Reynolds-number or the Grashof-number depending whether forced or free convection prevails. Therefore already in single phase flow scaling or similarity laws have a restricted and only approximate validity. This we have to keep in mind when we now discuss similarity in two-phase flow and in boiling heat transfer.

14.3 Dimensionless Numbers for Two-Phase Flow and Boiling Heat Transfer

From a theoretical point of view it is the best and most exact procedure to derive the dimensionless numbers from differential equations completely describing the interesting physical phenomenon by arranging the terms in a certain way. Another systematic procedure is to form a matrix from the exponents of

$$F = w^a \ ; \ \Delta p^b \ ; \ l^c \ ; \ \rho^d \ ; \ \eta^e$$

$$\left(\frac{m}{s}\right)^a \ ; \ \left(\frac{kg}{ms^2}\right)^b \ ; \ (m)^c \ ; \ \left(\frac{ka}{m^3}\right)^d \ ; \ \left(\frac{kg}{ms}\right)^e$$

mass (kg)	0	b	0	d	e ⋯
length (m)	a	-b	c	-3d	-e ⋯
time (s)	-a	-2b	0	0	-e ⋯
⋮	⋮	⋮	⋮	⋮	⋮

or

	a	b	c	d	e ⋯
kg	0	1	0	1	1 ⋯
m	1	-1	1	-3	-1 ⋯
s	-1	-2	0	0	-1 ⋯
⋮	⋮	⋮	⋮	⋮	⋮

Figure 14.1: Matrix for forming dimensionless groups.

the influencing parameters and their dimensions as indicated in Figure 14.1. The number of the influencing parameters minus the rank of the matrix gives the number of the dimensionless groups and these again can be elaborated by evaluating the matrix. Finally a third possibility to obtain dimensionless numbers is the method of trial and error by multiplying and dividing selected parameters.

By these methods or sometimes just by intuitive experience several dimensionless numbers were found for two-phase flow and for thermohydraulic processes with phase change already several years ago. Figure 14.2 gives some examples of them. They are usually restricted to a single phenomenon for example the Weber-number to droplet formation or the Jakob-number to boiling which represents the heat flux ratio in the liquid and in the vapour from the wall. A special role plays the Martinelli parameter X which was found by Martinelli studying two-phase pressure drop but which seems to have a much more general validity.

Weber number :

$$We = \frac{\rho \cdot d_d \ w^2}{G}$$

Laplace constant :

$$La = \frac{\delta}{g(\rho_l - \rho_g)} \ [m^2]$$

Jakob number :

$$Ja = \frac{c \cdot \rho_l (\vartheta_w - \vartheta_{bulk})}{h_{lg} \cdot \rho_g}$$

Bubble Reynolds number :

$$Re_b = \frac{\rho_g \cdot w_b \cdot d_b}{\eta_g}$$

Boiling number :

$$N_{Bo} = \frac{\dot{q}}{h_{lg} \cdot \rho_g \cdot w_g}$$

Buoyancy number :

$$N_{Bu} = \frac{\rho_l - \rho_g}{\rho_l}$$

Subcooling number :

$$N_{sub} = \frac{\Delta h_{sub} \ (\rho_l - \rho_g)}{h_{lg} \cdot \rho_g}$$

Phase change number :

$$N_{ph} = \frac{\dot{m}_g \cdot (\rho_l - \rho_g) \cdot l}{\rho_g \cdot \rho_l \cdot w_l}$$

Slip ratio

$$s = \frac{w_g}{w_l}$$

Drift number

$$N_D = \frac{w_g \ surface}{w_f \ surface}$$

Martinelli parameter :

$$X = \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \cdot \left(\frac{\eta_l}{\eta_g}\right)^{0.1} \cdot \left(\frac{1-\dot{x}}{\dot{x}}\right)^{0.9}$$

Figure 14.2: Examples for dimensionless numbers in two-phase systems.

In single phase and single component systems the velocity and the temperature field have to be regarded only but in two-phase flow in addition we have to take into account the density distribution - i.e. the void fraction or the quality - in the system. Finally in a multi-component system the concentration field would have to be added. In two-phase flow with and without boiling we must have information on

the velocity field,

the phase distribution field and

the temperature field.

The last one usually can be neglected if we have a single component two-phase system in thermodynamic equilibrium.

The second phase brings with it a lot of complications in our similarity or scaling deliberations. The hydrodynamic and thermodynamic properties in both phases have to be similar and also the phase change behaviour must be comparable. Before discussing whether these conditions are compatible we have briefly to think about, why and what do we want to scale in two-phase flow: In single phase flow usually we have the problem of scaling the size

of an apparatus or of a plant, that is we do the experiments with
smaller, scaled down objects. In two-phase flow there is the
question of the proportion too, but a much more severe problem
rises from the very large heat input, which is needed to do two-
phase and boiling experiments. One would need for example several
MW to make heat transfer experiments with a representative section
of a nuclear reactor fuel element. A geometrically scaling down
of the fuel rod array is almost not possible because the bubbles
keep their size and the ratio between bubble diameter and rod
space would change and so seriously influence the boiling phen-
omena. There is another possibility to reduce the power input by
using instead of water a modelling liquid with low latent heat
of evaporization. And so in two-phase flow not so much a scaling
of the geometry but a scaling of the fluid is of interest and
importance.

Due to the fact that thermodynamic properties of the gases
and the liquid phase of the modelling fluid play an important
role, it is essential that

(i)   these properties are well known and

(ii) that they show a similar behaviour as the original fluid.

Besides water the refrigerants (R11, R12, R13) are pretty well
researched and have well known thermodynamic properties. In
addition they offer convenient experimental conditions with respect
to pressure and temperature.

14.4   Thermodynamic Scaling of the Fluids

From a very first point of view the claim to the similarity
of the thermodynamic conditions seems simply to be solved by
selecting the thermodynamic state of the modelling fluid, in such
a way that the important influencing properties are comparable.
This can precisely be done for one property, but for the others
only approximatively. So we find very often in the literature
that in the scaling tests the thermodynamic state of the modelling
fluid is so selected that the density ratio ρ_f/ρ_g between liquid
and vapour is the same in the model and in the original. This
may be a very serious restriction because there can be other
properties as important as the density.

If there would be a common equation of state for both liquids
which we could write in a dimensionless reduced form with the
help of the thermodynamic consistency we could make use of the
thermodynamic theorem of corresponding states. One of the simplest
equations of state is certainly the Van der Waals-equation, which
we can write as follows:

$$\left(\frac{p}{p_c} + \frac{3}{v/v_c}\right)\left(3\,\frac{v}{v_c} - 1\right) = 8\,\frac{T}{T_c} \tag{14.4}$$

In this form - which is reduced with the critical data - the Van
der Waals-equation contains only universal constants not restricted
to a certain liquid. Certainly the Van der Waals-equation allows

only a very rough and unprecise prediction of the thermodynamic
properties of Freon. But it gives the hint that the critical data
p_c, T_c and v_c may be useful to get a corresponding state for
comparable thermal, caloric and even transport properties, since,
as Grigull (1968) showed there is according to the thermodynamic
consistency a strong familiarity between transport data and certain
caloric properties.

In a single component system the two phases usually only
exist along the saturation line which means that one thermodynamic
property fixes the whole state. So using the critical properties
we can either scale with the critical pressure ratio

$$\left(\frac{p}{p_c}\right)_{Original} = \left(\frac{p}{p_c}\right)_{Modell} \tag{14.5}$$

or with any other critical ratio like

$$\left.\frac{T}{T_c}\right|_0 = \left(\frac{T}{T_c}\right)_M \; ; \; \left(\frac{\rho}{\rho_c}\right)_0 = \left(\frac{\rho}{\rho_c}\right)_M \; ; \; \left.\frac{n}{n_c}\right|_0 = \left.\frac{n}{n_c}\right|_M \tag{14.6}$$

Comparisons have proved that the pressure ratio is the most
useful parameter for many applications especially for scaling
between water and Freon. As shown in Figure 14.3 there is only a
deviation of a few percent in the density ratio ρ_l/ρ_g of liquid
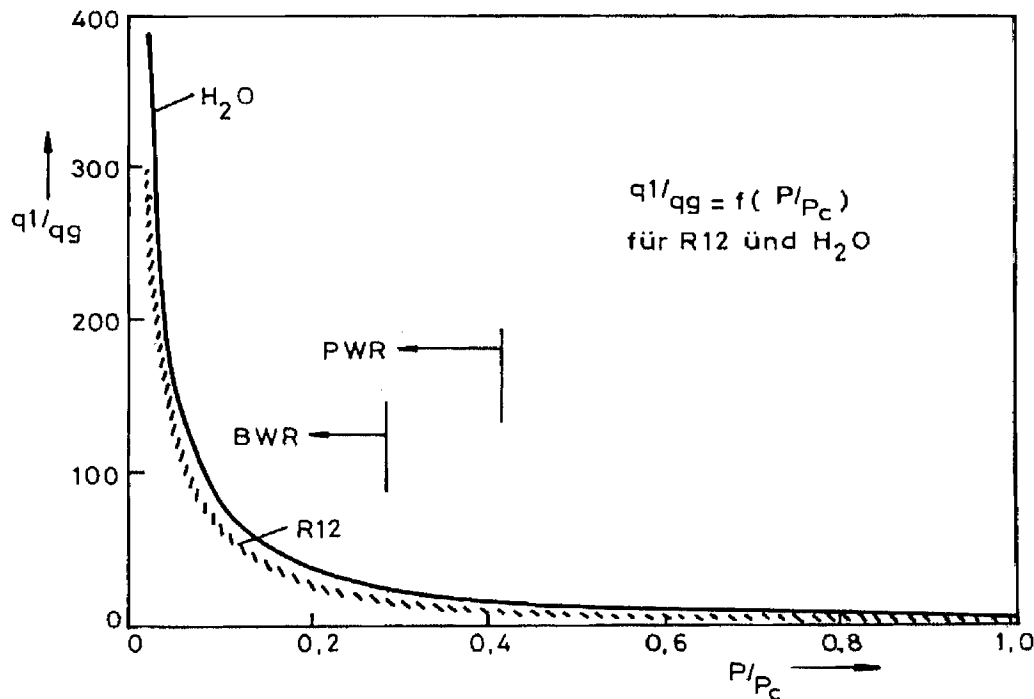and vapour for water and Freon if we scale it with the critical

Figure 14.3: Density ratio ρ_l/ρ_y of water and Freon 12 as a function of reduced
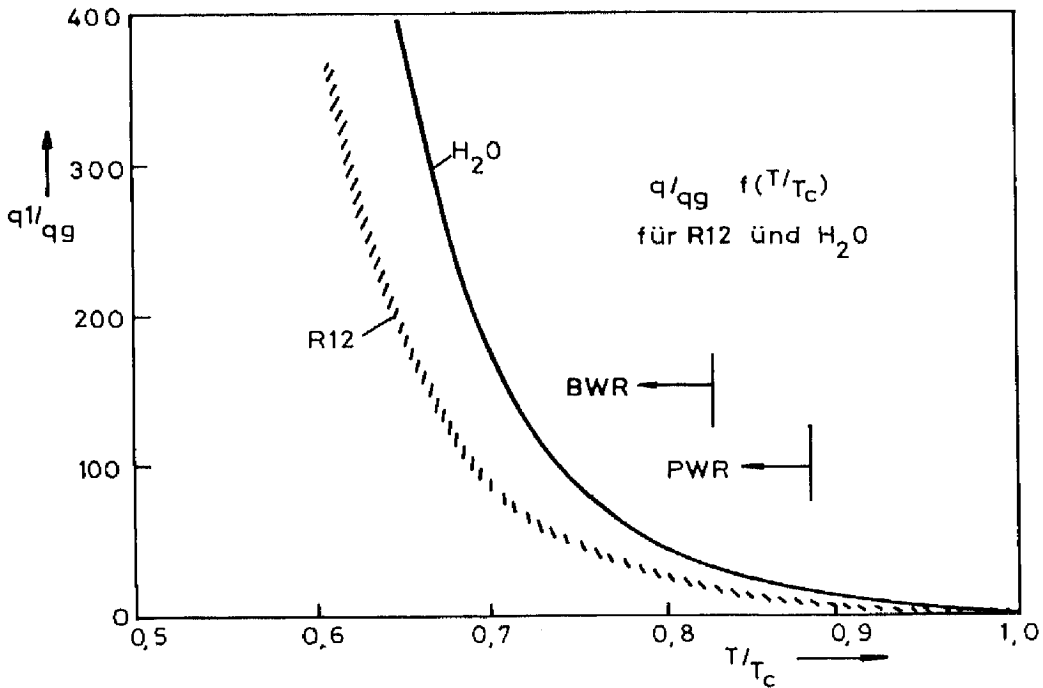pressure.

Figure 14.4: Density ratio ρ_l/ρ_g of water and Freon 12 as a function of reduced temperature.

pressure ratio. This again means scaling with p/p_c and with ρ_l/ρ_g give almost the same results. This is not the case if we use instead of the pressure ratio the critical temperature ratio as it can be seen from Figure 14.4.

In many hydrodynamic and heat transfer problems the density ratio may play the important role but there are other problems like bubble formation, flashing, entrainment and so on, where other properties as surface tension, viscosity, thermal conductivity are governing the phenomenon. A comparison of some properties scaled with the critical pressure ratio is given in Figure 14.5, where data for viscosity, thermal conductivity, surface tension, Prandtl-number and latent heat of evaporization are plotted. We see that the absolute values of these data for water and for Freon differ considerably. But we can transduce the Freon data to the water data by simply multiplying with an individual factor as demonstrated with the dotted lines in Figure 14.5. This factor then has to be taken in account also in the fluiddynamic scaling. If we deduce the mass flow rate for the modelling conditions with the help of the Reynolds-number for example we get the conidtion

$$ Re = \frac{\dot{m}d}{\eta} \; ; \; Re_0 = Re_M; \; \dot{m}_M = \dot{m}_0 \frac{\eta_M}{\eta_0} = k \cdot \dot{m}_0 \cdot \eta_M \tag{14.7} $$

where the factor k according to Figure 14.5 would be $k_l = 2.15$ and $k_g = 0.8$. In two-phase flow, even for this simple problem, we
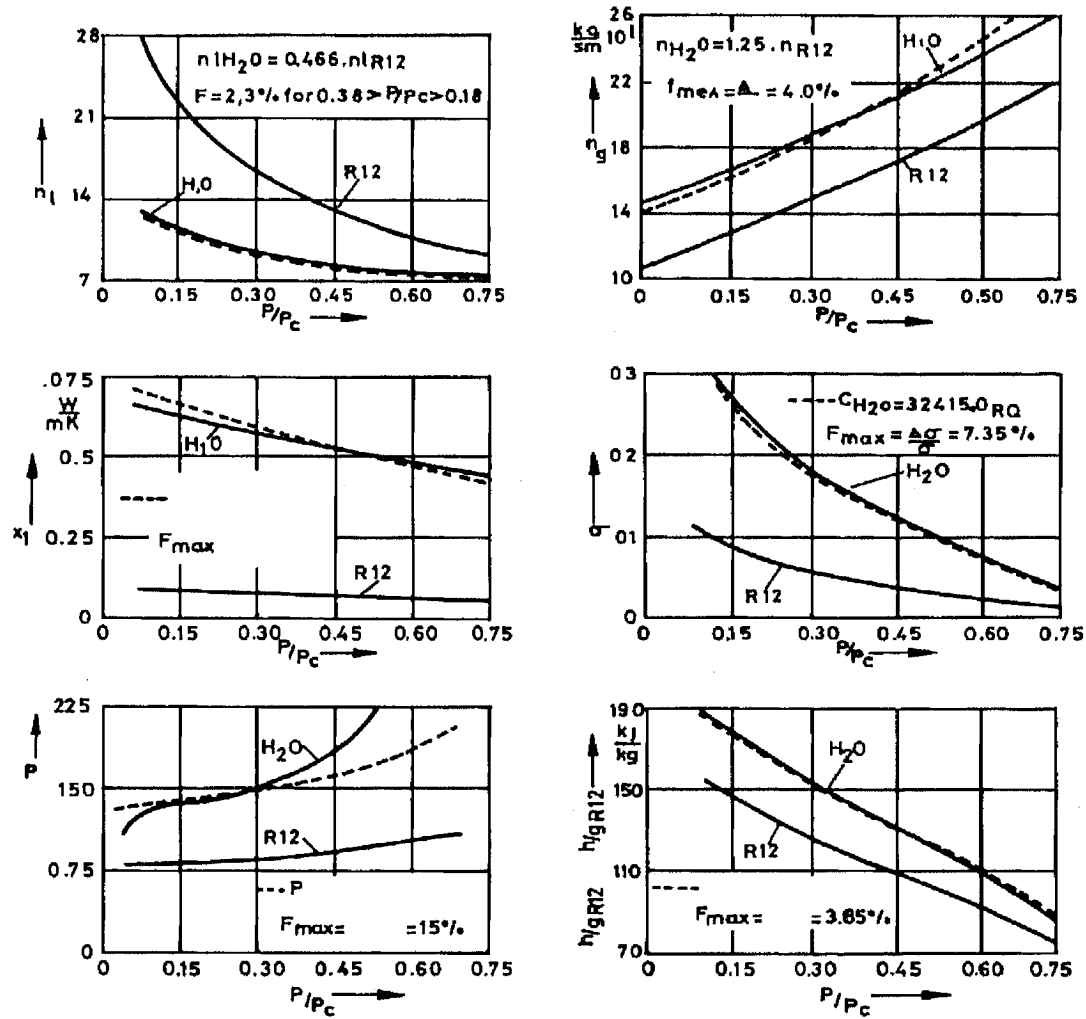
Figure 14.5: Comparison of thermodynamic and transport properties of water and Freon 12.

therefore have two correction factors one for the liquid and another one for the gaseous phase. One therefore has to decide which phase is the more important one for the special fluid dynamic process we are interested in. For scaling mass flow rate in two-phase flow we can primarily only use the viscosity either of the liquid or of the gas. For the ratio of liquid and vapour mass flow rate an additional condition exists namely the quality x for the void fraction ε. That means we have to neglect the influence of the viscosity of one of the phases. Already from this simple example we see that for two-phase scaling we have to subdivide in primary and secondary parameters to a much greater extent than we are used to doing in single phase flow.

In transient conidtions, for example with flashing in a sea-water desalination plant or during a loss of coolant accident in a water-cooled nuclear reactor, we need the first derivatives with respect to pressure of some thermodynamic properties in addition. As shown in Figure 14.6 also these derivatives can be made
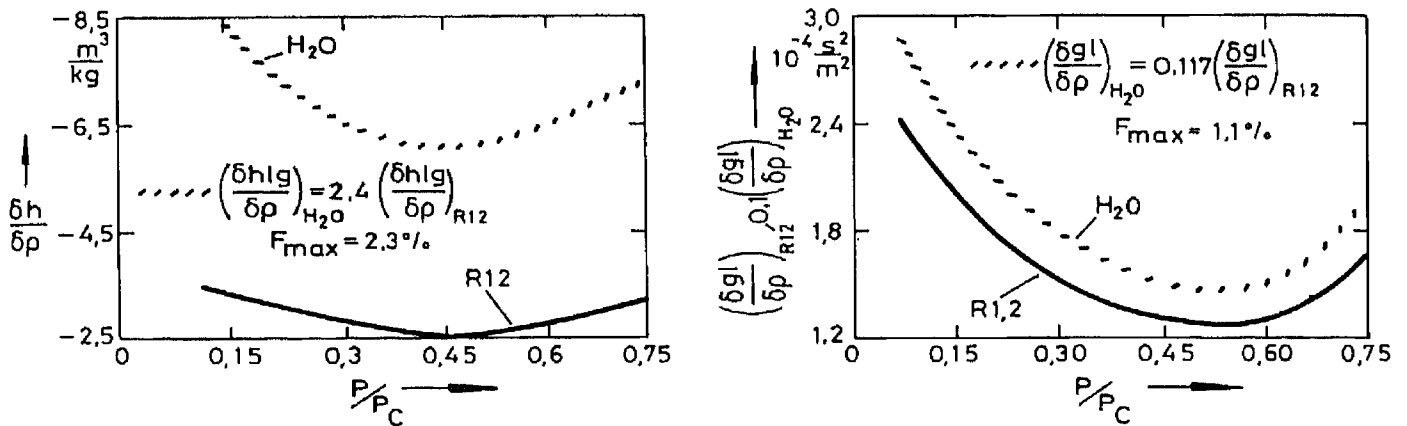
Figure 14.6: Comparison of derived thermodynamic properties (water and Freon 12).

coinciding by a simple multiplying factor.

A similar comparison of thermodynamic properties as discussed here but restricted to the surface tension and the viscosity can be found in a paper by Baker (1965) who introduced empirical reference values for surface tension and viscosity. A more detailed comparison is given in Belda's thesis (1975).

14.5 Scaling of Simple Fluid Dynamic Occurrences

In scaling we have to distinguish two steps, namely

(i)   to select the conditions in the mould and

(ii)   to transfer the results obtained in the test model to original conditions.

Under simple circumstances both steps can be made with the help of dimensionless numbers as well known for pressure drop and heat transfer problems in single phase flow. For more complicated cases the dimensionless numbers give us the approximate hydrodynamic and thermodynamic conditions for running the tests and the transfer has to be made by the help of a detailed mathematical description of the physical phenomena for example in the form of differential equations integrated in a numerical way. Let us first discuss examples of more simple fluid dynamic occurrences.

A first step in studying two-phase flow is the investigation of the flow pattern, i.e. the distribution of the two phases in any cross sections of the channel. Quandt (1963) deduced predictions for flow pattern from the deliberation, that the forces acting in the flow must be responsible for the phase distribution. If the forces induced by the pressure drop prevail, annular flow should be present and if the surface tension becomes greater than the forces resulting from the pressure drop and from the buoyancy there should be bubble flow. From these simple ratios of the forces we can derive dimensionless numbers as for example

$$N_{annular\ flow} = \frac{\rho\ w^2\ d}{\sigma} \qquad (14.8a)$$

$$N_{bubble\ flow} = \frac{\sigma}{\rho\ g\ d^2} \qquad (14.8b)$$

where equation (14.8a) is the ratio of the inertia and surface tension forces and equation (14.8b) is the ratio of the surface tension and the buoyancy forces. Equation (14.8a) is nothing else than the well-known Weber-number and equation (14.8b) represents the product of the Laplace-constant and a characteristic length for example the bubble diameter. Unfortunately the method of Quandt was not very successful in predicting the regimes of the different flow pattern. For horizontal flow another procedure for the prediction of flow pattern was proposed by Baker. He defined the two property parameters

$$\psi_{FB} = \left(\frac{\sigma H_2 O}{\sigma}\right)\left[\frac{\eta_l}{\eta_{H_2O}}\cdot\left(\frac{\rho H_2 O}{\rho_l}\right)^2\right] \qquad (14.9a)$$

$$\lambda_{FB} = \left[\left(\frac{\rho_g}{\rho_{air}}\right)\left(\frac{\rho_l}{\rho_{H_2O}}\right)\right]^{0.5} \qquad (14.9b)$$

and plotted a flow regime map with $1/\dot{x} = \dot{M}_l/\dot{M}_g$ and the mass flow rate of the gas as additional parameters, as shown in Figure 14.7. The terms in equation (14.9a) and (14.9b) are not dimensionless but we can regard them as a scaling help. For vertical flow often the flow regime map by Benett (1965) is used. Zetzmann (1976) demonstrated that in some cases simply by modifying the abscissa
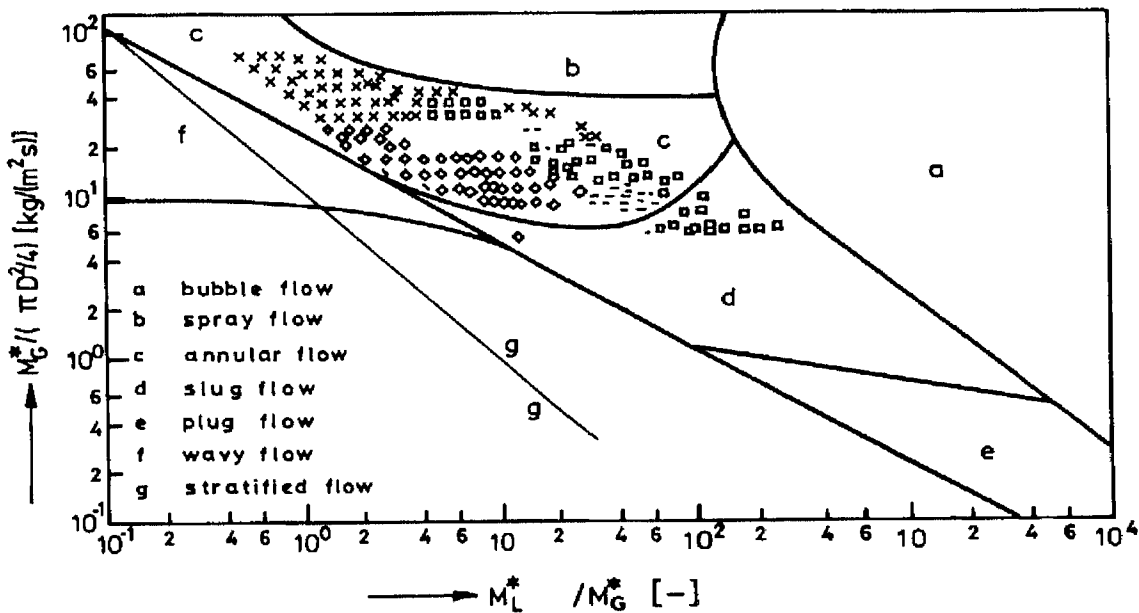

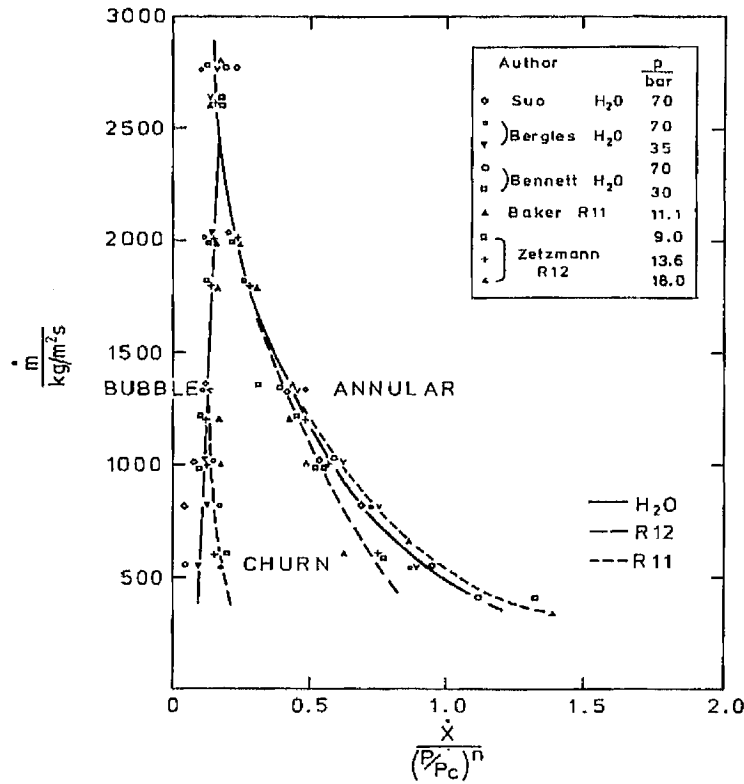Figure 14.7: Flow pattern map according to Baker (1965).

Figure 14.8: Bennett's flow pattern map, modified according to Zetzmann (1976).

of this diagram with the critical pressure ratio the flow regime map can be generalized for different fluids and pressures as shown in Figure 14.8.

A little more difficult to survey is the picture in the case of the void fraction ϵ and the slip ratio S which are closely connected if the quality $\dot{x}$ is given.

$$S = \frac{1-\epsilon}{\epsilon} \quad \frac{\dot{x}}{1-\dot{x}} \quad \frac{\rho_l}{\rho_g}$$

(14.10)

The slip ratio and through it the void fraction are influenced by the buoyancy forces and by the pressure drop in the channel. So empirical correlations for predicting void fraction, e.g. given by Kowalczewski (1964) or by Kütükcüglu (1967) contain the Froude-number

$$Fr = \frac{\dot{m}^2}{g \cdot d_h \cdot \rho_1^2}$$

(14.11)

as scaling parameter. Hewitt used the Martinelli-parameter

$$X = \sqrt{\frac{(\Delta p/\Delta l)_l}{(\Delta p/\Delta l)_g}} = \left(\frac{\rho_g}{\rho_l}\right)^{0.5} \cdot \left(\frac{\eta_l}{\eta_g}\right)^{0.1} \cdot \left(\frac{1-\dot{x}}{\dot{x}}\right)^{0.9} \tag{14.12}$$

for his void correlation. That means for doing modelling tests we would have to adapt either the Froude-number or the Martinelli-parameter. But both correlations do not claim general validity for a variety of fluids.

A first but very promising step for generalizing the prediction of void fraction was done by Nabizadeh (1976). Starting from the equation of Zuber and Findlay (1965).

$$\epsilon = \frac{\dot{x}}{\rho_g} \left[C_o \left(\frac{\dot{x}}{\rho_g} + \frac{1-\dot{x}}{\rho_l} \right) + \frac{1.18}{\dot{m}} \left(\frac{\sigma \cdot g (\rho_l - \rho_g)}{\rho_l^2} \right)^{0.25} \right]^{-1} \tag{14.13}$$

and supported by a large number of reliable own measurements he developed an empirical correlation for the c_o-factor in Zuber-Findlay's equation of the form

$$C_o = \left(1 + \frac{1-\dot{x}}{\dot{x}} \cdot \frac{\rho_g}{\rho_l} \right)^{-1} \left[1 + \frac{1}{n} \, Fr^{-0.1} \cdot \left(\frac{\rho_g}{\rho_l} \right)^n \cdot \left(\frac{1-\dot{x}}{\dot{x}} \right)^{\frac{11n}{9}} \right] \tag{14.14}$$

$$n = \sqrt{0.6 \, \frac{\rho_l - \rho_g}{\rho_l}} \tag{14.15}$$

Applying this extended and improved Zuber's-Findlay equation he gets quite good agreement between correlated and measured data for water, Freon 12 and Freon 113 as it can be seen from Figure 14.9. However, we cannot immediately deduce the parameters of a model test for given conditions of an original from equation (14.13) and (14.14) before looking to the magnitude of influence of the different terms. Nabizadeh points out that it is sufficient to adjust the same values of the density ratio of the quality and of the total mass flow density $\dot{m}$ in the model and in the original plant.

For pressure drop scaling very early a first step was done by Martinelli (1949). He gave a simple correlation between the two-phase multiplier ϕ_l^2:

$$\left(\frac{\Delta p}{\Delta l}\right)_{2ph} = \phi_l^2 \left(\frac{\Delta p}{\Delta l}\right)_{l \text{ only}}$$

$$\left(\frac{\Delta p}{\Delta l}\right)_{2ph} = \phi_g^2 \left(\frac{\Delta p}{\Delta l}\right)_{g \text{ only}}$$

$$\tag{14.16}$$

and the parameter X defined by himself and already mentioned in equation (14.12).
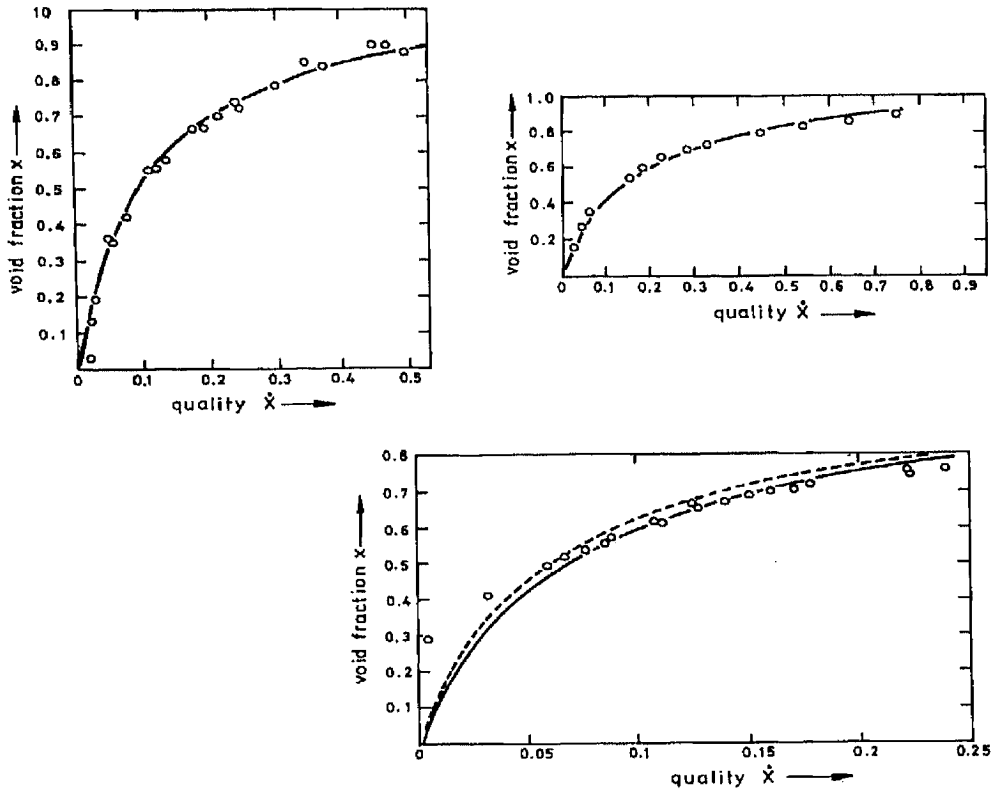
Figure 14.9: Void fraction, comparison of water and Freon data, scaled with
            Zuber-Findlay (1965) equation modified by Nabizadeh (1976).

In a detailed experimental and theoretical analysis Friedel
(1975) elaborated scaling rules for two-phase pressure drop. His
measurements prove that the two-phase multiplier, (equation
(14.16)) is the same in water and in Freon if the following
dimensionless groups are equal under test and under layout
conditions.

$$\left.\frac{\left(\eta_{1}/\eta_{g}\right)^{0.2}}{\left(\rho_{1}/\rho_{g}\right)^{0.9}}\right|_{0} = \left.\frac{\left(\eta_{1}/\eta_{g}\right)^{0.2}}{\left(\rho_{1}/\rho_{g}\right)^{0.9}}\right|_{M} \qquad (14.17)$$

$$\left.\frac{Fr_{1}}{Re_{1}^{0.25}}\right|_{0} = k \left.\frac{Fr_{1}}{Re_{1}^{0.25}}\right|_{M} \qquad \dot{x}\Big|_{0} = \dot{x}\Big|_{M} \qquad (14.18)$$

The second group with the Froude- and the Reynold-number contains
a scaling factor k which is a slight function of the pressure and
varies between 1.68 and 1.75. With good approximation for k the
value 1.7 can be taken. Friedel adapts the thermodynamic state
by choosing the same viscosity/density-ratio in the model and in

the original. Then the mass flow rate in the test is fixed by
the scaling factor k via the Reynold-number of the liquid.
Proceeding so measured data in water and in Freon fall along the
same line as demonstrated in Figure 14.10, where the two-phase
multiplier is plotted versus the quality $\dot{x}$.

Regarding heat transfer in two-phase flow we have to distinct
several regions

- a first one, in which bubbles are formed at the heated
  wall with low quality in the fluid,

- a second one with a thin liquid film at the wall and
  evaporization at the phase boundary between the liquid
  film and the gaseous bulk flow,

- a third one with a vapour film at the wall, which the
  liquid cannot wet and low quality in the fluid and

- a fourth one, where the liquid cannot wet the wall too,
  but where the fluid is mainly vapour containing a minor
  quantity of liquid in the form of droplets.

Heat transfer with bubble formation at the wall is a
function of the heat flux density and as a generalizing parameter
sometimes the Laplace-constant (see Figure 14.2) is used. The
whole bubble boiling process is up to now not well enough under-
stood to derive reliable scaling laws. A promising step to
describe heat transfer with nucleate pool boiling by using
dimensionless numbers

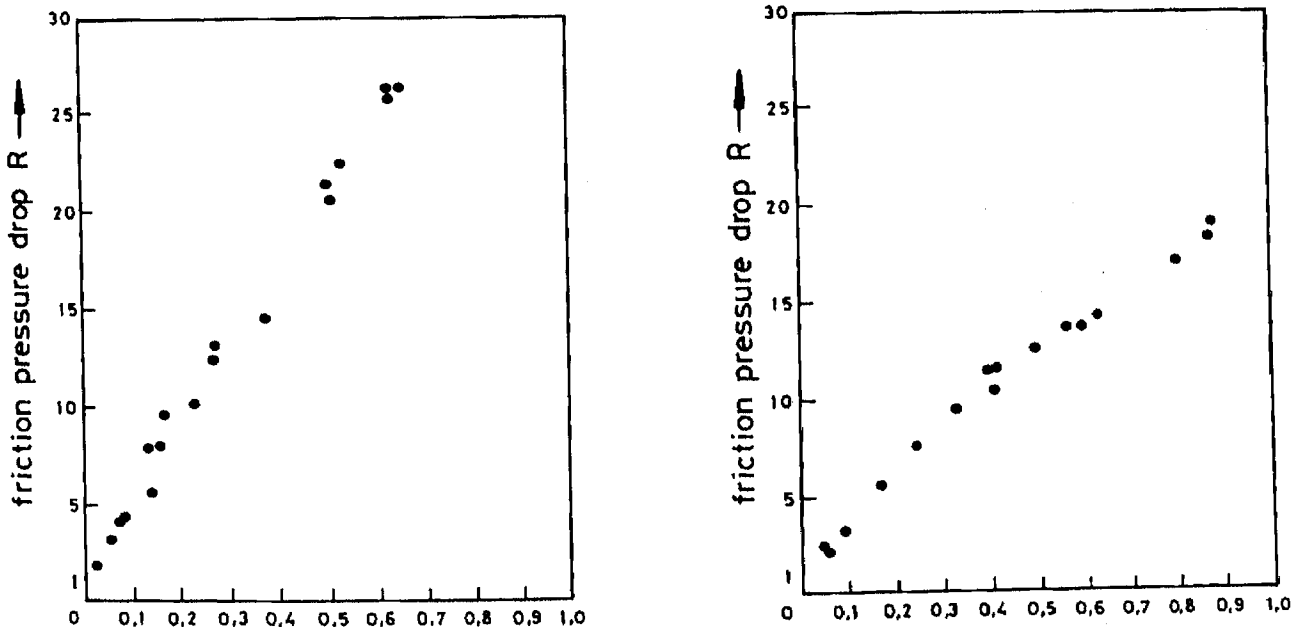$$Nu = \frac{\alpha \cdot d_{bub} \cdot}{\lambda_{1HC}} = 0.013 K_1^{0.8} \cdot K_2^{0.4} \cdot K_3^{0.133}$$

Figure 14.10: Two-phase flow pressure drop multiplier scaled according to
Friedel (1975).

$$K_1 = \frac{\dot{q} \cdot d_{bub}}{\lambda_{1HC} \cdot T_s} \quad ; \quad K_2 = \frac{d_{bub} \cdot T_x \lambda_{1HC}}{\sigma \cdot \nu_C} \tag{14.19}$$

$$K_3 = \frac{h_{1q} \cdot \rho_v \cdot K}{(f \cdot d_{bub})^2 \rho_1 \cdot d_{bub}}$$

f = bubble frequency   K = rel. roughness
                             of surface

was made by Stephan (1968), which also could be used for defining scaling criteria. In the high quality region the position is a little simpler with the thin liquid film at the wall through which the heat is transported by conduction and convection. Based on a proposal of Schrock and Grossmann (1962) the ratio of the heat transfer coefficients in two-phase flow and in single phase flow is given as a function of the Martinelli-parameter and of the boiling number.

$$\frac{\alpha_{2ph}}{\alpha_{1ph(liquid)}} = C_1 \left[N_{BO} \cdot 10^4 + C_2 \left(\frac{1}{x} \right)^n \right]^m \tag{14.20}$$

The heat transfer of the single phase-liquid flow can be calculated from the well known correlations in the literature usually containing the Reynold-number and the Prandtl-number. So scaling for this phenomenon should work if we select the Martinelli-parameter, the boiling-number and the quality correctly. But comparing heat transfer coefficients measured in Freon, organic fluid and water the empirical constants in equation (14.20) always have to be adjusted separately as Calus (1973) showed. Future experiments have to clarify whether there is an additional scaling parameter to be taken into account.

Heat transfer with spray- or fog-cooling, i.e. in the fourth above-mentioned region sometimes plays an important role in engineering practice as for example in the superheating part of a boiler or during the emergency core cooling of a nuclear reactor. Scaling these heat transfer conditions is a not too serious problem, because the heat transfer coefficient is mainly governed by the vapour flow and the liquid droplets are only cooling the boundary layer near the wall and so improving the heat transport. Dimensionless groups for modelling therefore are a modified Reynolds-number and the Prandtl-number related to the vapour phase. A well-known correlation is the equation of Dougall-Rohsenow.

$$Nu = 0.023 \, Re^{0.8} \, Pr_g^{0.4}$$

$$Re = \frac{\dot{m} d_h}{\eta_g} \left[x + \frac{\rho_g}{\rho_1}(1-x) \right] \tag{14.21}$$

Scaling criteria are the Reynolds-number as defined in equation
(14.21), the Prandtl-number of the vapour, and the quality of
the fluid. Comparisons with experimental data measured in water
and Freon gave quite good results and good agreement with
Dougall-Rohsenow's correlation. From this example we learn that
always this phase, which governs the fluiddynamic or heat
transfer process has to be adjusted in its thermodynamic
properties.

In some cases also for problems with interfacial momentum
and heat transfer at the phase boundaries unique correlations and
such scaling criteria can be found. Viecenz (1980) studied the
phase separation, the drift flux and the mean void fraction $\langle \epsilon \rangle$
in liquid pools with gas or vapour bubbling through. He
correlated the volume averaged void fraction $\langle \epsilon \rangle$ by

$$\langle \epsilon \rangle = C \left[\frac{u_o^{"2}}{g\sqrt{\frac{\sigma}{g(\rho'-\rho'')}}} \right]^n \left[\frac{\sqrt{\frac{\sigma}{g(\rho'-\rho'')}}}{d_{Beh}} \right]^{0.176}$$

$$\left[\frac{\rho'}{\rho'-\rho''} \right]^{0.585} \cdot \left[\frac{v'}{v''} \right]^{0.255}$$

(14.22)

where C and n depend from the superficial velocity $u_o^{"}$ of the gas
in the vessel (assuming no liquid in the vessel) and the Laplace
constant as shown in Figure 14.11. This figure also proves that
there is good agreement between the predicted results by equation
(14.22) and measurements in the literature. Equation (14.22)
represents a simple and reliable scaling law in the form of
dimensionless numbers for phase separation in gasified liquids.

14.6   Scaling Combined Phenomena

The fluiddynamic behaviour in an engineering apparatus like
a boiler, a nuclear reactor or destillation and extraction
columns is a very strongly combined effect of several simple
phenomena and usually fluid flow and heat transfer are linked
together. The heat transferred to the fluid determines the void
fraction, the produced vapour influences, the pressure drop,
which governs the mass flow rate and this again can effect the
heat transfer. Up to now we regarded only each single simple
phenomenon separately assuming no influence from other processes.

Out of the numerous problems of technical two-phase flow
for discussion here we select the burnout behaviour and the blow-
down conditions during a loss of coolant accident in a nuclear
reactor.

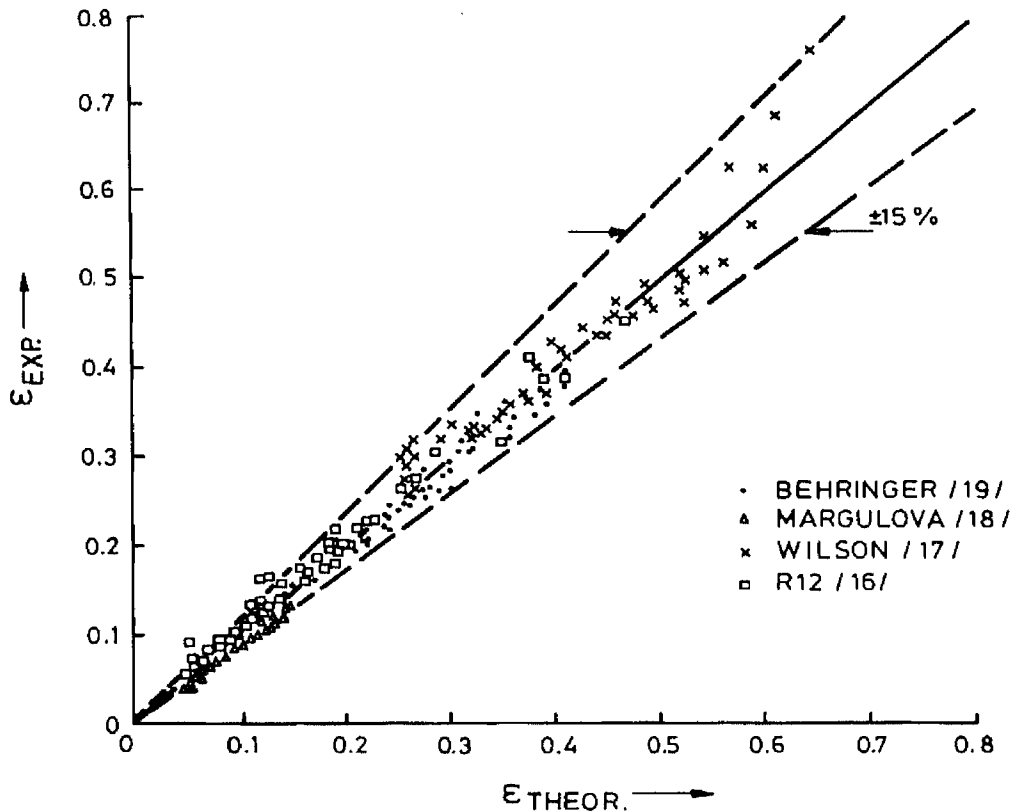The exact knowledge of the critical heat flux with boiling

Figure 14.11: Comparison of experimental data with equation (14.22).

(burnout or dryout) is an urgent problem in the layout of the
fuel elements of water cooled nuclear reactors. Due to the high
heat input and the extensive instrumentation the experiments
are very expensive. Therefore it was obvious to look for more
convenient and cheaper test conditions. Here Freon as a modelling
fluid seemed to offer a possibility. There are several proposals
in the literature for scaling critical heat flux from Freon to
water. All start from the thermodynamic condition that the
density ratio of liquid and vapour has to be the same in the
modelling and in the original fluid.

Stevens (1965/66) (1970) preferred a more empirical
procedure and made a direct comparison of the burnout data in
Freon and water with simple scaling factors. He postulated that
in addition to the same density ratio and the identical geometry
the quality (or subcooling) at the inlet and the outlet of the
heated length have to be the same in the model and in the
original. Therefore only the mass flow rate remains as free
variable to be adjusted in the scaled down conditions. This
scaled mass flow rate is got via the scaling factor K by plotting
the quality $\dot{x}_A$ (at the outlet) versus the parameter

$$\left[\dot{m} \ d_h^{0.25} \left(\frac{d_h}{l} \right)^{0.59} \right] \ ;$$

as this is done in Figure 14.12 with the help of measurements in
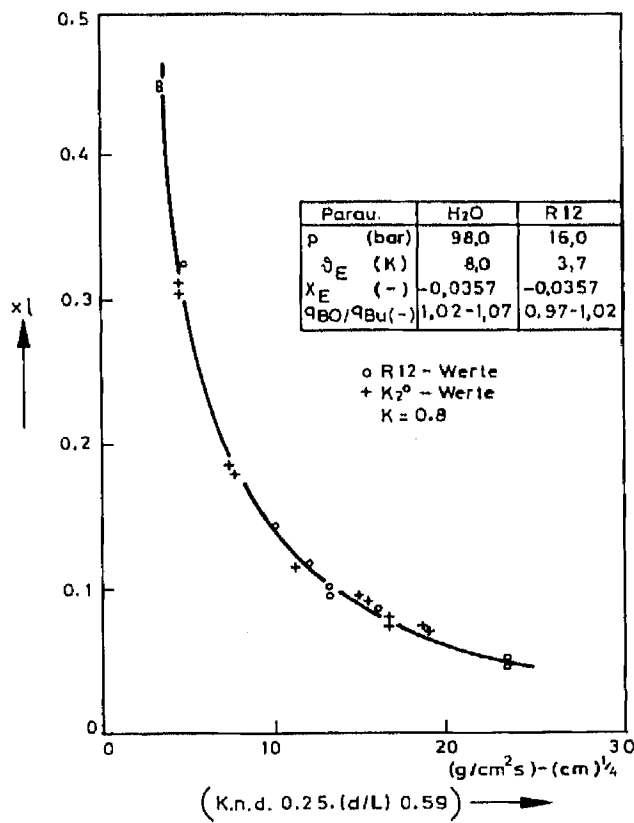a rod cluster which were carried out by Hein and Kastner (1972).

Figure 14.12: Stevens' (1965/66) burnout scaling factor for mass flow.

Barnett (1963) used the dimensionless analysis for deriving his scaling laws. He listed all parameters, which are according to the status of knowledge mainly governing the physical behaviour of the phenomenon and others which have a minor influence. Doing so, he gets a relation of the form

$$q_{CHF} = f(d, l, \dot{m}, p, \Delta h, h_{lg}, \rho_l, \rho_g c_l, \lambda_l, \nu_l, \sigma, \frac{d(\rho_l/\rho_g)}{dp}) \qquad (14.23)$$

generally describing the critical heat flux. Adapting the dimensional analysis equation (14.23) can be arranged in the form of dimensionless groups and he gets

$$\frac{q''}{h_{lg}^{1.5} \rho_g} = f \left(\frac{1}{d}, \frac{D \cdot \rho_g \cdot h_{lg}^{0.5} c \cdot p_l}{\lambda_{1_{HE}}}, \frac{\dot{m}}{\rho_g h_{lg}^{0.5}}, \frac{\Delta h}{h_{lg}}, \frac{\rho_l}{\rho_g}, \right.$$

$$\left. \frac{\nu \cdot \rho_g \cdot c_{pl}}{\lambda_{1_{HC}}}, \frac{\sigma \cdot c p_l}{h_{lg}^{0.5} \cdot \lambda_1}, h_{lg} \cdot \rho_g \cdot \frac{d(\rho_l/\rho_g)}{dP_s} \right) \qquad (14.24)$$

$$q'' = f\ (l, d, \dot{m}, p, \Delta h, h_{lg}, \rho_l, \rho_g, c_{pl}, \lambda_l, v_{l}, \sigma, \frac{d\,(\rho_l/\rho_g)}{dp_s})$$

$$\gamma = \frac{d\,(\rho_l/\rho_g)}{dp_s} \qquad \beta = \frac{d\vartheta_s}{dp_s}$$

No	properties	equation $q'' = f(l, d, \dot{m}, p, \Delta h)$	F_q	$F_{d,l}$	$F_{\dot{m}}$	$F_{\Delta h}$
4	$h_{lg}, \rho_l, \rho_g, c_{pl}, \lambda_l, \gamma$	$\dfrac{q''\cdot\gamma^{0.5}}{\lambda\sqrt{\rho_l}} = f\ (\dfrac{l}{d}, \dfrac{d\cdot c_{pl}\cdot\sqrt{\rho_l}}{\lambda_l\cdot\gamma^{0.5}}, \dfrac{\dot{m}\cdot\gamma^{0.5}}{\sqrt{\rho_l}}, \dfrac{\rho_l}{\rho_g}, \dfrac{\Delta h}{h_{lg}})$	22,43	0,717	1,914	11,72
5	$h_{lg}, \rho_l, \rho_g, \beta, \lambda_l, \gamma$	$\dfrac{q''\cdot\gamma^{0.5}}{h_{lg}\sqrt{\rho_l}} = f\ (\dfrac{l}{d}, \dfrac{d\cdot h_{lg}\cdot\gamma^{0.5}\sqrt{\rho_l}}{\beta\cdot\lambda_l}, \dfrac{\dot{m}\gamma^{0.5}}{\sqrt{\rho_l}}, \dfrac{\rho_l}{\rho_g}, \dfrac{\Delta h}{h_{lg}})$	22,43	0,482	1,914	11,72
10	$h_{lg}, \rho_l, \rho_g, \mu_g, \gamma$	$\dfrac{q''\cdot\gamma^{0.5}}{h_{lg}\sqrt{\rho_l}} = f\ (\dfrac{l}{d}, \dfrac{d\cdot\sqrt{\rho_l}}{\mu_g\cdot\gamma^{0.5}}, \dfrac{\dot{m}\cdot\gamma^{0.5}}{\sqrt{\rho_l}}, \dfrac{\rho_l}{\rho_g}, \dfrac{\Delta h}{h_{lg}})$	22,43	0,814	1,914	11,72
12	$h_{lg}, \rho_l, \rho_g, c_{pg}, \lambda_g, \beta$	$\dfrac{q''\sqrt{\beta\cdot c_{pg}}}{h_{lg}^{1.5}\cdot\rho_l} = f\ (\dfrac{l}{d}, \dfrac{d\cdot\sqrt{c_{pg}\cdot h_{lg}\cdot\rho_l}}{\lambda_g\cdot\sqrt{\beta}}, \dfrac{\dot{m}\cdot\sqrt{\beta\cdot c_{pg}}}{\sqrt{h_{lg}\cdot\rho_l}}, \dfrac{\rho_l}{\rho_g}, \dfrac{\Delta h}{h_{lg}})$	2 308	0,317	1,968	11,72

Figure 14.13: Scaling factors for critical heat flux according to Barnett
          (1963).

From these groups in equation (14.24) Barnett developed several scaling factors N_n depending which thermodynamic properties he was taking in account in addition to the values of latent heat of evaporization h_{lg} and density of liquid and vapour ρ_l, ρ_g.

Examples of sets for scaling factors elaborated by Barnett with this method are shown in Figure 14.13. Barnett offers several possibilities for scaling critical heat flux depending which combination of thermodynamic properties is assumed to be the most appropriate one.

Bouré (1970) was the first, who clearly stated that a comprehensive scaling of the critical heat flux is not possible and that only a restricted similarity of the boiling crisis phenomenon in water and in Freon may exist. Realizing this he emphasized his study to elaborate strict rules for a limited similarity.

He distinguishes - similar as Barnett - two groups of parameters: primary parameters and secondary ones. He used the later ones to determine correction factors for the primary parameters. Doing so he obtained the following 4 dimensionless groups.

$$N_1 = K_1\ \frac{\rho_l}{\rho_g}\ ;\quad N_2 = K_2\ \frac{\Delta h_{inl}}{h_{lg}}$$

$$N_3 = K_3\ \frac{q\cdot l}{h_{lg}\cdot\dot{m}\cdot d} \qquad N_4 = K_4\ \frac{\dot{m}}{\rho_l\ g_l}$$

(14.25)

A: SINGLE TUBE P = 137·3 BAR
B: 9 ROD BUNDLE P = 96·5 BAR
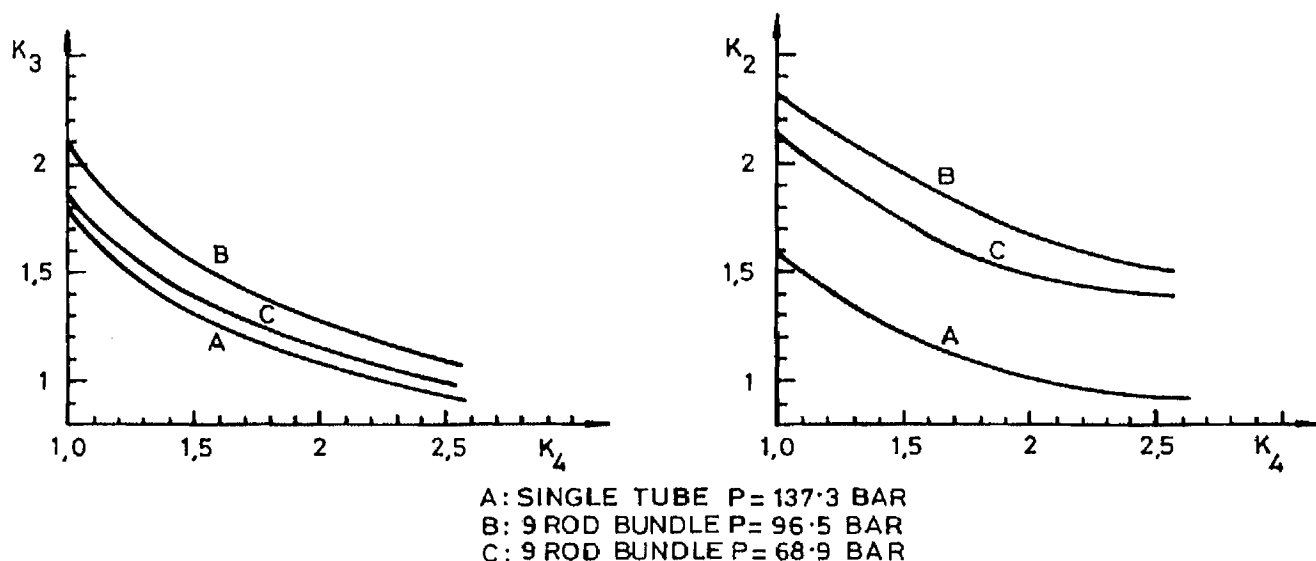C: 9 ROD BUNDLE P = 68·9 BAR

Figure 14.14: Scaling factors for critical heat flux, according to Bouré.

The correction- or scaling factors K_1-K_4 are depending from the geometry of the channel as well as from the pressure. The values of two of them can be chosen arbitrarily and by using the same density ratio of liquid and vapour in water and in Freon the factor K_1 becomes unit. The scaling factor K_4 may vary in the order from 1 to 2.5 and by fixing this factor the scaling factors K_2 and K_3 can be taken from Figure 14.14.

Finally we have to mention the method proposed by Ahmad (1971) who also subdivided the parameters in groups, namely system describing, primary and secondary parameters. He gets 13 dimensionless numbers from which he selects 3 for restricted scaling arranging them as a function of the modelling parameter

$$\psi = f\left(\frac{\dot{m} \cdot d}{\eta_l} , \frac{\eta_l}{\sigma \cdot d \cdot \rho_l} , \frac{\rho_l}{\rho_g}\right)$$ (14.26)

In addition he gives a function for the modified boiling number (see Figure 14.2)

$$N_{BO}^{*} = \frac{\dot{q}}{\dot{m} \cdot h_{lg}} = f\left(\psi, \frac{\Delta h_{sub}}{h_{lg}} , \frac{\rho_l}{\rho_g} , \frac{1}{d}\right)$$ (14.27)

and from experiments he empirically finds the following relationship between the scaling numbers:

$$\psi = \left(\frac{\dot{m} \cdot d}{\eta_l}\right)\left(\frac{\eta_l}{\sigma \cdot d \cdot \rho_l}\right)^{n_1}\left(\frac{\rho_l}{\rho_g}\right)^{n_2}$$ (14.28)

Rearranging the terms in equation (14.28) we realize that the modelling parameter ψ is a function of well-known dimensionless numbers

$$\psi = \underbrace{\left(\frac{\dot{m}^2 \cdot d}{\rho_l \cdot \sigma}\right)^{0.67}}_{We} \cdot \underbrace{\left(\frac{\dot{m} \cdot d}{\eta_l}\right)^{-0.133}}_{Re_l^*} \cdot \underbrace{\left(\frac{\dot{m} \cdot d}{\eta_g}\right)^{-0.2}}_{Re_g^*} \qquad (14.29)$$

So in addition to Barnett's and Bouré's energy orientated dimensionless groups Ahmad introduced hydrodynamic scaling numbers.

Before we check the validity of these scaling laws for critical heat flux we have to remind that the uncertainty of burnout measurements in in the order of $\pm$ 10%. The best agreement between scaling and original data can be found for simple geometries of course. With inside cooling uniformly heated tubes agreement within $\pm$ 7% was realized. More interesting for practical use but also more complicated due to additional influencing hydrodynamic parameters is the scaling of critical heat flux for rod bundles, Hein and Kastner (1972) compared own measurements in water and Freon and scaled them according to Bouré as shown in Figure 14.15. They found that there is an influence of the mass flow rate on the factor K_3 (see equation (14.25)), which scales the critical heat flux from Freon to water. The tests in Figure 14.15 were done with a 9-rod cluster. The influence of the mass flow rate is relatively small if all 9 rods
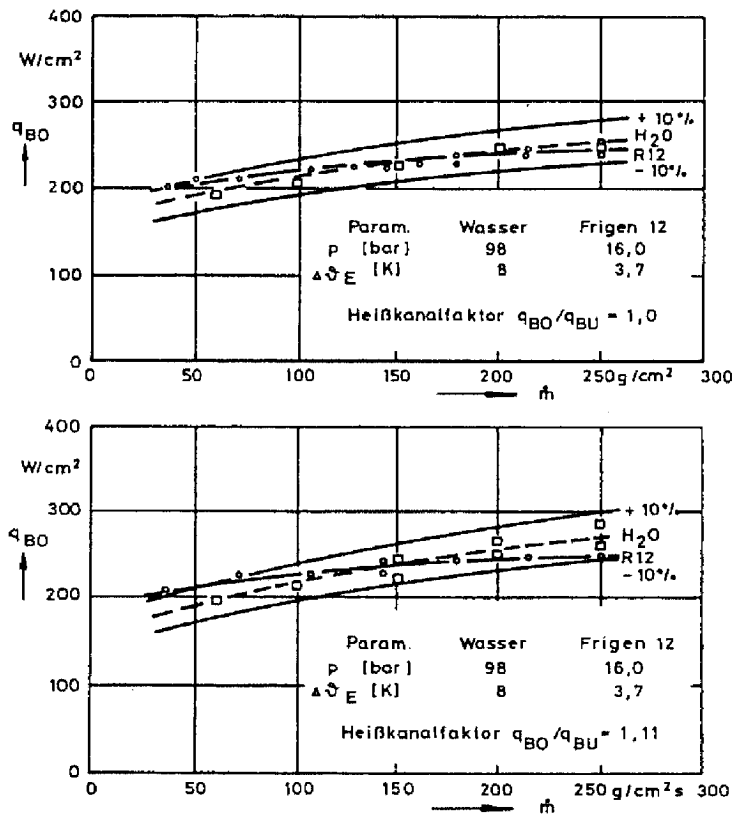

Figure 14.15: Influence of mass flow rate and radial power distribution on critical heat flux scaling (Bouré).

400
W/cm²

⊙ R12-Werte, $F_q = 14,59$
○ R12-Werte, $F_q = 14,59 \cdot (\frac{\dot{m}}{\dot{m}_k})^{0,13}$
□ H₂O-Werte,

300

q_{CHF}

200

Param.	Wasser	Frigen 12
p (bar)	98	16,0
$\Delta\vartheta_E$ (K)	8	3,7

Grod bundle
Uniform heat flux

100

0

0   50   100   150   200   250   300 g/cm²s
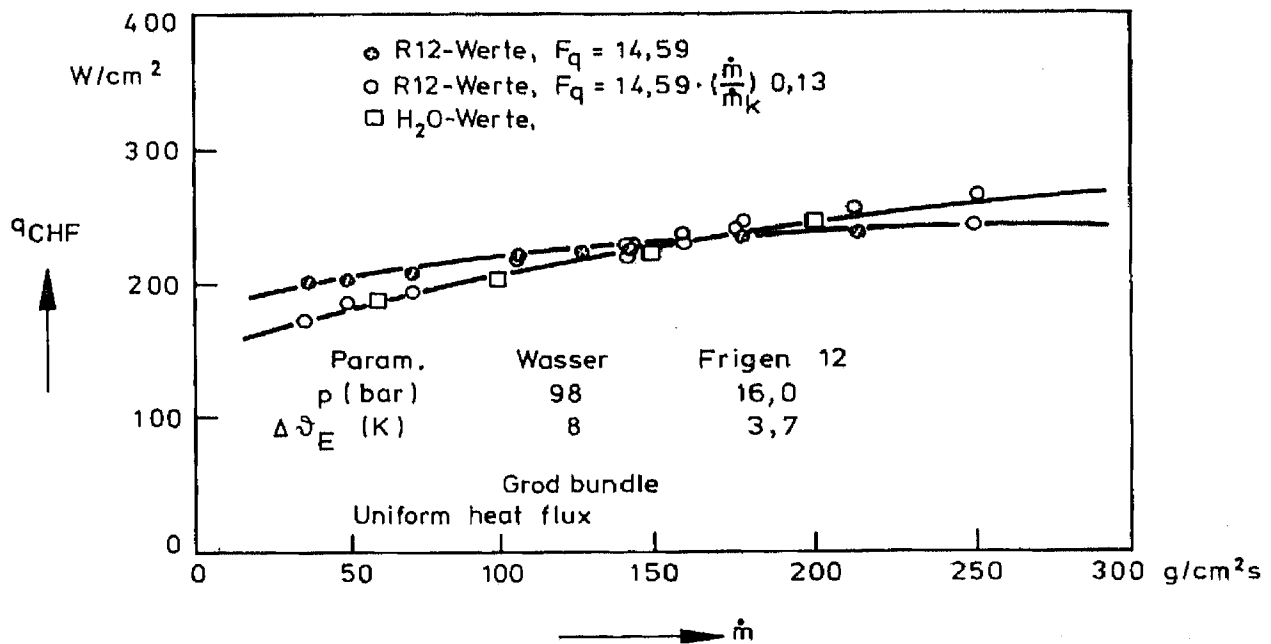
$\dot{m}$

Figure 14.16: Mass flow correction for critical heat flux scaling (Bouré, Hein).

have the same heat flux, i.e. if there is uniform heat flux distribution. With increasing hot channel factor the deviation of the scaling values become greater, which is probably due to a mixing effect between the subchannels not being taken into account in the scaling deliberations. This is a clear hint for the restrictive similarity.

For uniform heat flux distribution water and Freon data can be easily brought in agreement by a simple mass flow dependent correcting function proposed by Hein and Kastner (1972) as demonstrated in Figure 14.16. It has to be mentioned that in this comparison of correction taking into account the pressure dependency of the scaling factor, which was proposed later by Courtaud and De Bousquet is not yet considered. For more detailed information on the agreement beween scaling and measured data of critical heat flux it may be referred to a paper by Hein (1972) (1976).

From the comparison in Figure 14.15 we learned that hydrodynamic phenomena like mixing may have a strong influence on critical heat flux scaling. The first reaction on this problem may be to simulate all hydrodynamic conidtions as good as possible, i.e. to use exactly the same hot channel factor in the water and in the Freon test. This, however, presupposes, that the similarity conditions for mixing and for critical heat flux are the same else we can only scale one of both correctly. From this we can draw the conclusion, that - generally spoken - scaling with dimensionless numbers only is not sufficient in two-phase flow. We need a theoretical description of the whole thermodynamic and fluiddynamic process, predicting the behaviour in the model as well as in the original set-up.

The circumstances become even more complicated if we consider

transient conditions for example the thermohydraulic behaviour of the two-phase flow during a blowdown due to a loss of coolant accident in a reactor. Doing experimental research work for investigating the fluiddynamic and heat transfer aspects of this problem we are forced to scale even using water in our experimental set-up, because the rod bundle in the test can only be a very small section of a whole reactor core, i.e. we have a geometrical scale down. The number of rods tested is usually limited by the total power input available to heat them. If we use Freon instead of water we can multiply the number of rods by a factor of approximately 15. But then we have to ask how to scale from Freon to water in addition.

Blowdown tests with Freon were made in our laboratory and compared with blowdown measurements performed with water by a German manufacturer of nuclear reactors (KWU, 1974). The conditions in the Freon loop were very carefully adapted and scaled to the water test rig. There was a two fold purpose of these measurements namely to study the burnout delay time during blowdown and to determine the heat transfer coefficient in post-dryout conditions. Depending which of these two phenomena are intended to scale one gets different similarity terms. Here we just want to discuss a little more detailed the dryout delay scaling.

The blowdown occurrence is too complex to overcome the scaling problems simply by using dimensionless numbers. One has to develop a physical conception of the thermohydraulic events and formulate it in mathematical expressions as differential equations. Due to the rapidly reduced pressure flashing and then a very high acceleration directed to the break occurs in the fuel elements. In case of a large break in the primary system of a pressurized water reactor the pressure would be reduced within about 20 s from 160 to approximately 5 bar. Emphasis has to be given to imitate in the Freon loop these very severe flashing conditions and the resulting acceleration of the fluid. The acceleration can be guaranteed by adapting the same local and temporal pressure ratio, which can be done by lowering the outside pressure of the Freon loop to about 0.1 bar. Then we still have the problem of critical (sonic) mass flow rate and different heat storage conditions in the Freon and in the water loop. All these problems can only be overcome by a detailed theoretical analysis used as basis for scaling. This analysis has to predict the results of Freon tests and the water tests as well. The scaling is not done from one experimental condition to the other but both test results are compared with theoretical predictions.

The tests were done for boiling water reactor conditions, where vapour is present already in steady state operation. The liquid phase is flowing along the heated wall and the vapour is concentrated in the bulk between the rods. The dryout, i.e. the disappearance of the liquid layer at the wall then is caused by the following effects:

(1)   Evaporization (heat flux from the rods)

(2)   Flashing due to pressure decrease

(3)   Entrainment due to high vapour velocity

$$\frac{d}{dz}\left[(\rho_l w_l (1-\epsilon)(1-e) + \rho_l w_E \cdot (1-\epsilon)\cdot e)\right] + \frac{d}{dz}(\rho_g w_g \epsilon) + \frac{d}{dt}\left[(\rho_l(1-\epsilon)(1-e) + \rho_l(1-\epsilon)\cdot e + \rho_g \epsilon)\right] = 0$$

$$\frac{1}{h_g}\frac{d}{dz}\left[(\rho_l w_l(1-\epsilon)(1-e) + \rho_l w_E \cdot (1-\epsilon)\cdot e)\right] + \frac{d}{dz}(\rho_g w_g \epsilon) + \frac{1}{h_g}\cdot\frac{d}{dt}\left[(\rho_l(1-\epsilon)(1-e) + \rho_l(1-\epsilon)\cdot e)h_l + (\rho_g \epsilon h_g)\right] = \dot{q}(z)\cdot\frac{U}{A_T}\cdot\frac{1}{h_g}$$

$$\dot{M}_l = \rho_l \cdot A_T \cdot (1-\epsilon)(1-e)\cdot w_l$$

$$\dot{M}_E = \rho_l \cdot A_T \cdot (1-\epsilon)\cdot e \cdot w_E$$

Figure 14.17: Mass and energy equation for dryout delay during blowdown.

Velocities of phases

$$w_{ph} = \sqrt{(\frac{dp}{dz})_{ph}\cdot \frac{2d_h}{(\xi\cdot\rho)_{ph}}}$$

$$(\frac{dp}{dz})_{ph} = \frac{1}{\phi^2_{ph}}\cdot(\frac{dp}{dz})_{2-ph}$$

$$\frac{1}{\phi_l^2} = (1-\epsilon)^2 \qquad \frac{1}{\phi_g^2}\cdot\frac{\epsilon^{2.5}}{1+75\cdot(1-\epsilon)}$$

$$w_l = \sqrt{\frac{A_l}{A_T}\cdot(\frac{dp}{dz})_{2-ph}\cdot\frac{2d_h}{\xi_r \rho_l}}$$

$$w_g = \sqrt{\frac{\sqrt{A_T\cdot(A_T-A_l)}}{A_T - 75A_l}\cdot(\frac{dp}{dz})_{2-ph}\cdot\frac{2d_h}{\xi\cdot\rho_g}}$$

Figure 14.18: Dryout delay during blowdown, equations describing phase velocities.

We can describe the whole procedure with the well-known constitution equations for mass, energy and momentum. In addition one needs information on the slip ratio because the difference in the phase velocity produces entrainment.

Belda (1975) developed an analysis on this basis starting from the equations given in Figure 14.17 and using for the phase velocities the equations derived from pressure drop deliberations as pointed out in Figure 14.18. The entrainment behaviour during the transient process was tried to be predicted by aid of investigations carried out by Hewitt (1970). With this mathematical treatment he gets a differential-integral equation describing the film thickness as a function of time and place.

$$\frac{1}{\rho_l\cdot l}(\dot{M}_l(l,t)-\dot{M}_l(0,t)+\dot{M}_E(l,t)) + \frac{\partial A_l}{\partial t} + A_l(\frac{1}{\rho_g}\cdot\frac{\partial P_l}{\partial t} +$$

$$\frac{1}{h_{lg}}(\frac{\rho_g}{\rho_l}\cdot\frac{\partial h_g}{\partial t}\cdot\frac{\partial h_l}{\partial t}) + A_{ges}(\frac{\int \dot{q}(z)\cdot dz\cdot U_b}{\rho_l\cdot h_{lg}\cdot l\cdot A_{ges}} - \frac{\rho_g}{\rho_l}\cdot\frac{1}{h_{lg}}\cdot\frac{\partial h_g}{\partial t}) = 0 \qquad (14.30)$$

In equation (14.30) the local and temporal values of mass flow rate and quality in the fuel element or in the test section have to be known from blowdown calculations, which can be performed for example with the well-known computer codes like RELAP IV or BRUCH. We then can determine from equation (14.30) at what time after the break the liquid layer at a certain position of a rod

in the bundle would become zero, which means that the dryout
occurs. For adapting the Freon tests conditions one can derive
from equation (14.30) and from the equations in Figure 14.17
and 14.18 dimensionless scaling numbers, too. They are listed in
Figure 14.18. From this figure we can see that the system pressure
is four times over determined. We therefore have to decide,
which of these four expressions is the dominant one. It can be
easily seen that the dependence of the liquid enthalpy from
pressure is much smaller than the change of latent heat of
evaporization with pressure. So two of the conditions can be
ruled out and the remaining two fortunately give quite similar
values in the order of 0.19. But there is a 5th conditioning because
we decided to scale with the critical pressure ratio and from this
we get the value of 0.188. So finally 0.19 could be chosen as a
good compromise.

By scaling the hydrodynamic and thermodynamic conditions in
the Freon loop before and during the blowdown with the dimension-
less numbers given in Figure 14.19 we got a quite good agreement
between water and Freon behaviour as can be seen from Figure
14.20, where the temporal course of system pressure, mass flow
rate and void fraction and the pressure drop in the fuel element
are plotted for water and for Freon versus the blowdown time.
This conformity, however, does not mean that the dryout delay
time are the same in Freon and in water because we have only made
the hydrodynamic side of the process analogous and we could not
completely scale thermodynamic details for example stored heat in
the construction materials and the complete interference between
thermodynamic and hydrodynamic properties. This means we have to
do an indirect scaling by comparing the measured data with the
theoretically predicted ones. As Figure 14.21 demonstrates there
is good agreement between measured and predicted dryout delay
time. The comparison was done for different locations of the
break - in the hot and in the cold leg of the primary system -
for a variety of break areas and for different heat flux densities
in the moment before the accident occurs.

Starting from fundamental deliberations the theoretical
approach should predict the behaviour in water as well as in
Freon. Unfortunately there are available only very few water
tests where the dryout delay time during blowdown was measured
and where in addition the temporal course of the hydrodynamic
conditions in the fuel elements is reliably known. Comparison
with water experimental data at hand show good agreement with the
theory as can be seen from Figure 14.22. This encourages to draw
the conclusion that scaling on the detour of a theoretical
analysis is a promising procedure for combined and complicated
two-phase flow phenomena.

To support this comparison between measured Nusselt-numbers
in water and Freon during blowdown may be shown in Figure 14.23
without discussing the scaling laws for this special problem. In
this figure measurements done with water (KWU, 1974) and with
Freon (Viert & Mayinger, 1979) are compared with correlations.
Keeping in mind, that there are many uncertainties arising from
the measuring technique the agreement is satisfactory. It has to
be added, that the comparison can be made only in the period
between 5 and 12 s after the break appears, because before we

heat flux

$$K_1 = \frac{\dot{q}(z) \cdot l \cdot U_B}{m_{GES} \cdot A_{GES} \cdot h_{lg}}$$

pressure

$$K_{21} = \frac{p_s}{\rho_l} \cdot \frac{\partial \rho_l}{\partial p} \qquad K_{23} = \frac{p_s}{h_{lg}} \cdot \frac{\rho_g}{\rho_l} \cdot \frac{\partial h_l}{\partial p}$$

$$K_{22} = \frac{p_s}{h_{lg}} \cdot \frac{\partial h_l}{\partial p} \qquad K_{24} = \frac{p_s}{h_{lg}} \cdot \frac{\partial h_{lg}}{\partial p}$$

acceleration

$$K_3 = \frac{\Delta p \cdot \rho_l}{\dot{m}^2_{GES}} = Eu_l$$

$$K_4 = \frac{\Delta p \cdot \rho_g}{\dot{m}^2_{GES}} \cdot \frac{\rho_g}{\rho_l} = Eu_g \cdot \frac{\rho_g}{\rho_l}$$

mass flow rate

$$K_5 = \zeta_l \triangleq Re = \frac{\dot{m}_{GES} \cdot d}{\eta_l}$$

geometry

$$K_6 = \frac{\sqrt{A_{GES}}}{l} \triangleq \frac{d_h}{l}$$

Figure 14.19: Dimensionless groups for dryout delay scaling during blowdown (Belda, 1975).
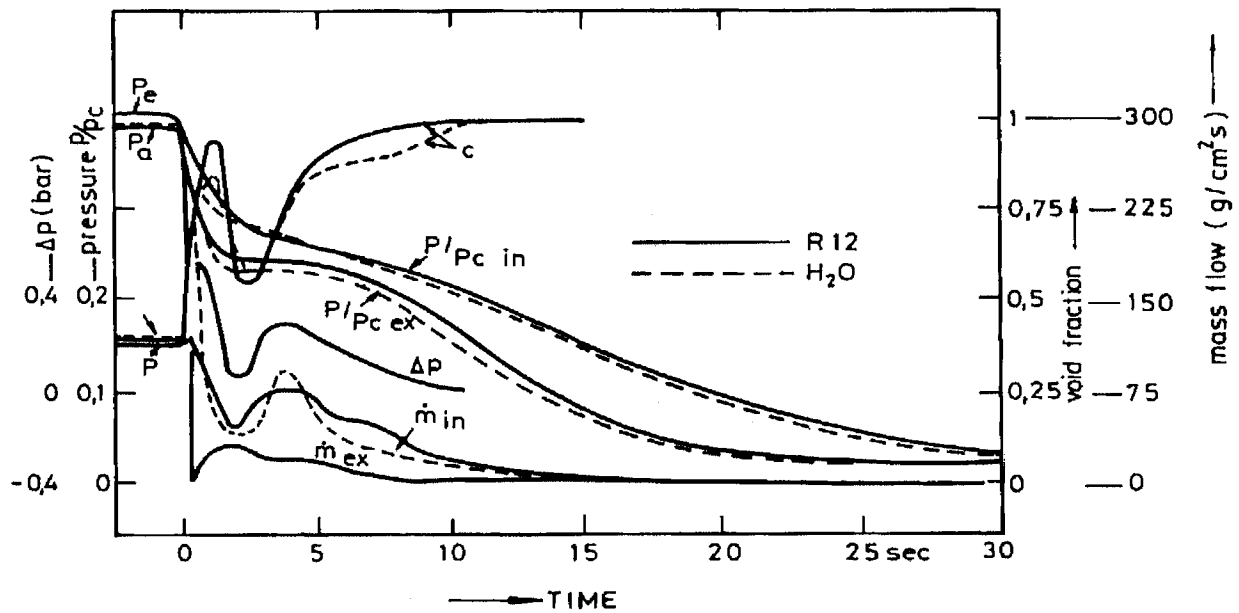

Figure 14.20: Blowdown during loss of coolant accident, comparison between water tests and scaled Freon 12 data.
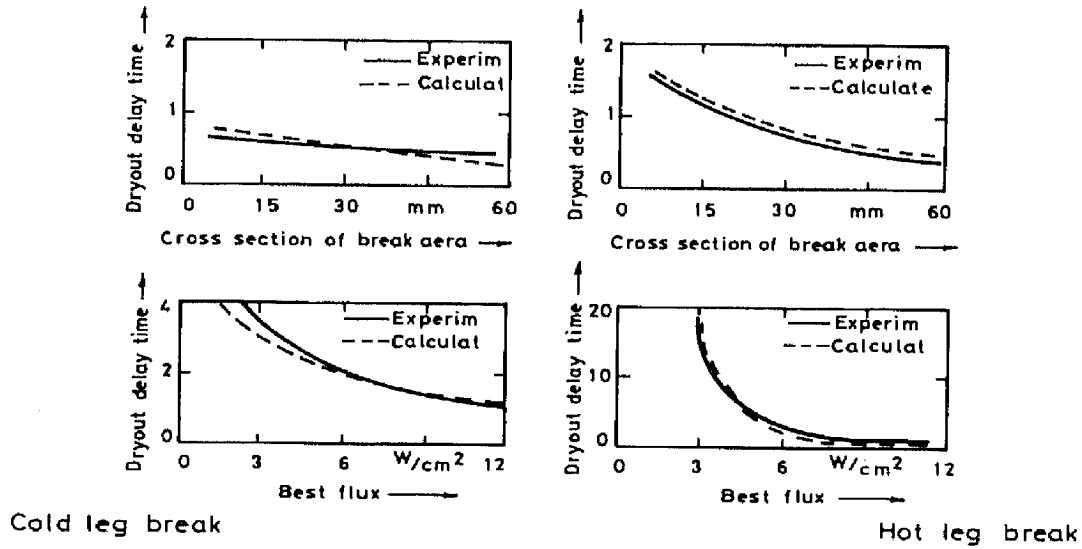
Figure 14.21: Dryout delay time, measured (in Freon 12) and calculated data.

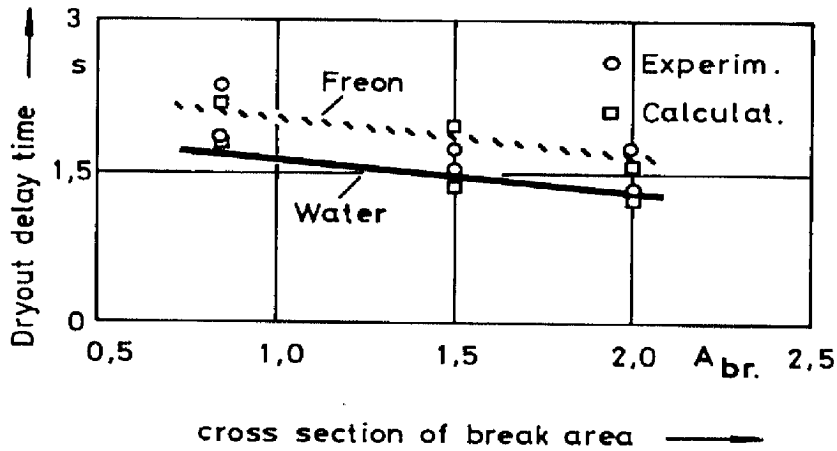cross section of break area ————

Figure 14.22: Dryout delay time, comparison of experimental and theoretical
             data for water and for Freon 12.

have transition boiling for which the used scaling model is not
valid and afterwards the hydrodynamic conditions in the water and
in the Freon loop differ considerably.

    There is certainly a lot of other complicated two-phase flow
problems which are of great technical interest. I should mention
the phenomena of subcooled boiling, of flow oscillations and
instabilities and the interchannel mixing.

14.7 Conclusion

    Even in single phase flow scaling is restricted under certain
circumstances as we have seen in the example of combined forced
and free convection. This limitation becomes much stronger in
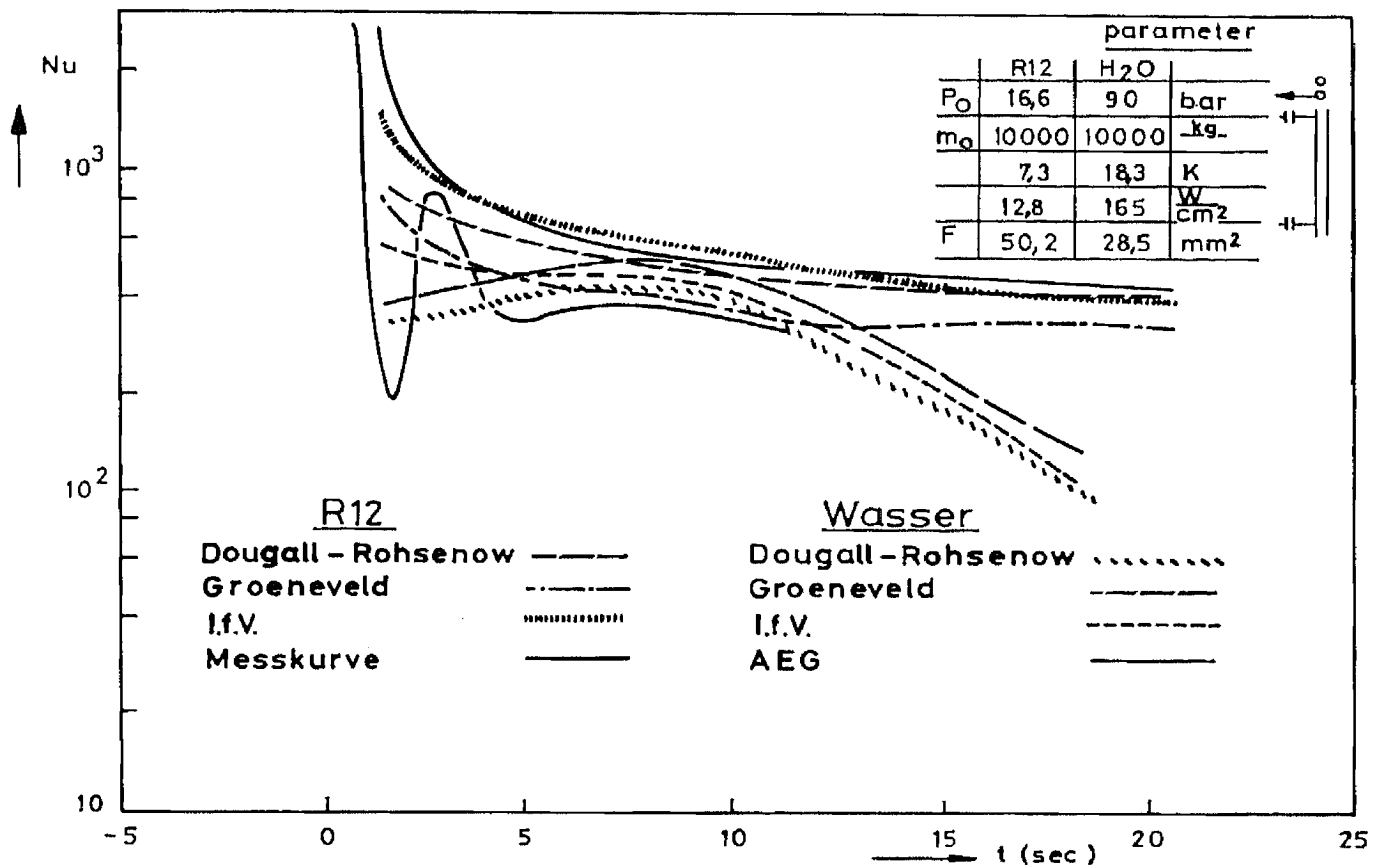two-phase flow. Whilst in a single phase fluid we have usually

Figure 14.23: Heat transfer in post dryout conditions during blowdown, comparison of water data and scaled Freon data.

similarity for hydrodynamic and heat transfer processes simultaneously, in two-phase flow each modelling law or scaling number is only valid for a single special phenomenon.

Scaling only with the help of dimensionless numbers is limited in two-phase flow to simple and isolated problems, where the physical phenomenon is a unique function of a few parameters. If there is a reaction between two or more physical occurrences, dimensionless scaling numbers mainly serve for selecting the hydrodynamic and thermodynamic conditions of the modelling tests and we have to separate the influencing parameters in primary ones determining the system and secondary ones which are of minor influence.

In not too complicated cases scaling to the original circumstances can be done by empirical correlations considering the important physical laws of the process. For many technical applications in future we shall be forced to scale via a computer code which analyses and describes the thermohydraulic phenomena as good as possible.

This makes it necessary to get a better understanding of the thermohydraulic behaviour in two-phase flow systems. A better

theoretical approach would also help us to generalize experimental
data and such to make full use of the numerous experimental results
in two-phase flow, which up to now stand to a great extent apart
and uncomparable. So scaling is not only an expedient to save
expenses for experimental research work, it also helps to compare
and generalize measured data. Elaborating scaling laws for two-
phase flow we have to put more effort to the theoretical analysis
of this process.

Nomenclature

c	- specific heat
d	- diameter
g	- acceleration of gravity
h	- enthalpy
h_{lg}	- latent heat of evaporation
l	- length
$\dot{m}$	- mass flow rate
M	- mass flow
p	- pressure
$\quad p$	- pressure drop
$\dot{q}, q''$	- heat flux
s	- slip
T	- temperature
v	- volume
$\dot{V}$	- velocity
$\dot{x}$	- quality
x, y, z	- length coordinates

Greek symbols

ψ_{FB}	- flow pattern number		η	- viscosity
α	- heat transfer coefficient		ϕ	- two-phase parameter
ε	- void fraction		ρ	- density
λ_{FD}	- flow pattern number		σ	- surface tension
λ_{HC}	- heat conductivity		ν	- kinematic viscosity
			ν	- temperature

Dimensionless groups

Fr	- Froude-Number
Gr	- Grashof-Number
N	- bubble-number
N_{BO}	- boiling-number

Nu   — Nusselt-Number
Pr   — Prandtl-Number
Re   — Reynolds-Number

Subscripts

c   — critical
CHF   — critical heat flux
g   — gas
h   — hydrodynamic
H_2O — water
ṁ   — inlet
l   — liquid
lg   — evaporation
m   — model
o   — original
s   — saturation
sub   — subcooling

References

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