Subcooled Boiling

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1. Introduction

The practical layout of heat exchangers and steam generators especially in former times was only based on the assumption that boiling does not start before the mean temperature in the liquid has reached its saturation point, that is the beginning of the first bubble formation was calculated by an energy balance according to the first law of thermodynamics. This assumption is not allowable at high heat flux densities as they are present for example in oil-fired boilers, in heat exchangers with great temperature differences between the primary and the secondary fluid but especially in the core of water-cooled nuclear reactors.

At high heat flux densities vaporazation may occur at the heated surface in spite of the fact that the cooling liquid did not yet reach the saturation point in its mean temperature. This phenomenon is called "subcooled boiling" which is due to a thermodynamic non-equilibrium in the liquid. There is a superheating of the liquid in the boundary layer near the heated wall, whilst the bulk temperature is still fairly subcooled. Bubbles formed near the wall at first grow due to heat and mass transfer from the superheated boundary layer but then by further enlarging their volume or by movement they come beyond the superheated boundary layer and start to condense again where by now the heat transfer is reversed.

Regarding a cooling channel with high heat-addition where subcooled liquid enters, in the literature usually as shown in figure 1 four zones are distinguished.

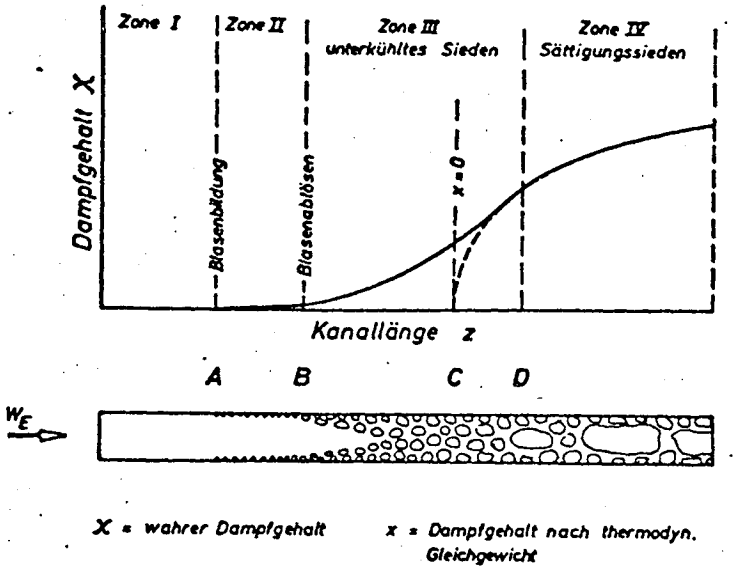

Fig. 1: Boiling regions in subcooled flow boiling

In the zone I due to the high subcooling of the liquid there is pure singlephase flow. The heat transfer from the wall to the fluid takes place by forced convection only. Due to heat addition the temperature of the cooling liquid raises and eventually the wall temperature reaches a point at which on technical rough surfaces bubble formation starts, that is the boiling nuclei become active but the subcooling of the fluid is still high enough that the bubbles cannot grow far beyond the very thin superheated boundary layer, and they are recondensed immediately as soon as their top enters the subcooled zone. Looking at this phenomenon one gets the impression that the bubbles adhere at the wall. In this zone II the steam content or the void is very low and is regarded as a pure wall effect.

Flowing along the channel the difference between saturation temperature and bulk temperature of the fluid becomes smaller. The bubbles formed in the superheated boundary layer can now detach from the heated surface and slowly condense downstream in the subcooled bulk. From this point of the bubble detachment - marked with B in figure 1 - the void in the zone III increases rapidly. The fluid is as mentioned not in thermodynamic equilibrium because in the subcooled bulk there are a lot of vapor bubbles also having a short duration of life. Further downstream finally the fluid reaches thermodynamic equilibrium and then we have pure net steam production.

This phenomenon of subcooled boiling for the present indeed represents a compensating safety potential if the layout of the heat exchanger is insufficient, because the heat transfer conditions are improved when boiling starts and thus the local temperature differences are diminished. However, the bubble formation due to subcooled boiling can result in very severe disturbences of the flow even in a breakdown, due to the fact that, as well known, the two phase flow has a much higher pressure drop than the single phase one. Not or not sufficiently regarding these boiling phenomena there is the danger that for example the layout of the pump for a loop is insufficient or that flow instabilities may occur, i.e. periodical changes in the mass flow rate.

For predicting the beginning of boiling and the real quality or void in heat exchangers and steam generators with high heat flux densities there are several theoretical models which all involve many empirical assumptions. Here for the beginning I just want to mention the models and theoretical treatments of Bowring /1/, Levy /2/, Lavigne /3/, Tong /4/ and Rouhani /5/. The uncertainty of these theoretical models mainly results from

1. empirical assumptions regarding the beginning of boiling, that is the first bubble formation
2. the mathematical description of bubble formation and bubble growth and
3. the physically correct description of bubble condensation.

All these processes are a function of the temperature decay in the boundary layer near the wall and at the phase boundary between the bubbles and the liquid. For a better understanding it seems therefore suitable to discuss the peculiarities of bubble formation in subcooled liquids first.

2. Bubble formation and bubble collapse in subcooled liquids

As well known there is a great number of publications describing the boiling phenomena in saturated liquids and today there is general understanding and agreement that bubbles start from roughnesses, that is small cavities in the heated wall. The heat stored as latent heat in the vapor of the bubble is attained there on a detour from the superheated liquid boundary. But the bubble does not only carry latent heat in the vapor, in addition, due to the high turbulence by detaching from the wall, it transports heat in the liquid drift flow behind it. Regarding the heat and mass transfer between the superheated boundary layer and the forming vapor bubble there are a lot of theoretical and also experimental works which cover a wide field of assumptions starting from pure heat conduction in the liquid boundary up to high convective eddies produced by differences in surface tension over the bubble circumference. One may mention for example the papers by Forster and Zuber /6/, Han and Griffith /7/, Plesset and Zwick /8/, Beer /9/ and Winter /10/. Reliable statements about the real behaviour need measuring techniques of highest accuracy. The conventional measuring devices very often are too slow or even influence the process in a non-permissable extent. Endeavoring to get more information about the subcooled boiling phenomena we used besides the well-known microthermocouples a new optical method, the so-called holographic interferometry in our laboratory at the Technical University Hannover. It would take too much time to describe this measuring technique in detail, therefore, here only a few results worked out with this method and by the help of high-speed cinematography should be shortly pointed out.

In spite of long years of research work in bubble boiling even under saturated conditions as figure 2 shows there are very different statements about bubble formation especiall· bubble growth.

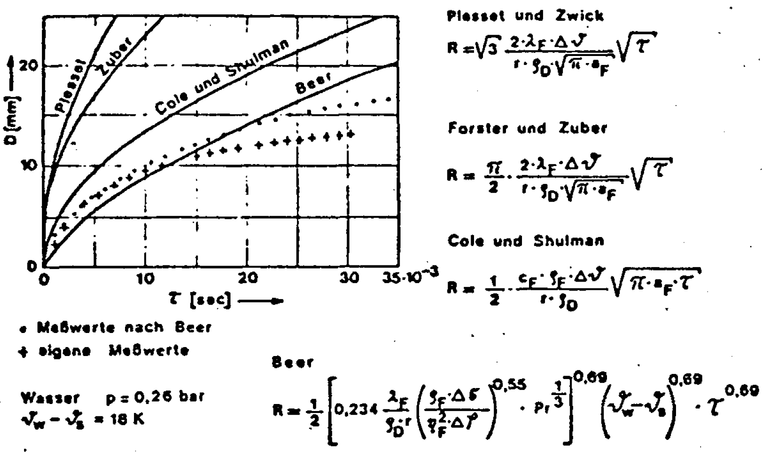

Fig. 2: Bubble growth in saturated pool boiling

Recent calculations, as for example by Beer /9/, who takes in account the so-called Marangoni-effect, influencing the heat and mass transfer at the phase boundary, seems to give better results. Whereas the bubble at saturated conditions reaches asymptotically its equilibrium, in subcooled boiling very early the condensating process starts sometimes even super-imposing on the growing behaviour. Due to this the prediction of bubble formation and bubble collapse is very difficult.

As we shall see from holographic pictures taken with a high-speed camera, the superheated boundary layer at the heated wall is at the beginning formed relatively slowly, and suddenly there grows a bubble out of it, which then depending on the liquid temperature outside the superheated boundary layer recondenses again. The condensation process primarly starts at the top of the bubble. By this a very violent circulation round the bubble is produced and then the heat exchange prevails at the lower parts of the bubble due to liquid eddies formed by the bubble movement. Partially one can also see during the condensation a compression and extension process, which may result in a superheating or subcooling of the vapor in the bubble. This results in a pulsating and very unstable bubble collapse.

The growing of a bubble takes about one tenth of the time needed for the following condensation. In principal one can use for the condensation similar equations as for bubble growth, and there are given several formulations in the literature, which mainly differ in treating the heat transfer at the phase boundary and in considering the inertia effects. They all assume spherical shape of the bubble and uniform conditions over the bubble circumference. In figure 3 the results of some mostly new theories are presented and compared with measured results.

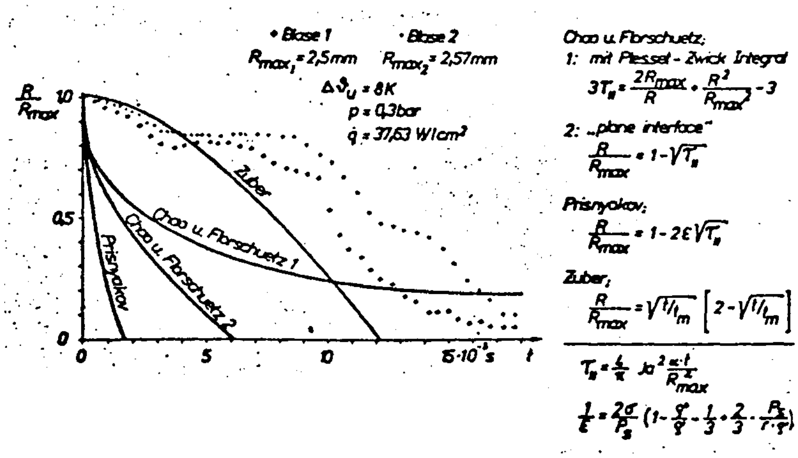

Fig. 3: Bubble condensation in subcooled pool boiling
Theoretical and experimental results

As one can see there is not the best agreement. The dynamic behaviour of the condensation can be seen best if one calculates the first and second derivation from the growing and collapsing curves. One then gets the growing and collapsing velocities and also the acceleration. This was done in figure 4 for the above mentioned example, the bubble growth and collapse was measured in this example, as mentioned by a high-speed-camera and one can see from the measuring points that there seems to be an oscillating behaviour of the bubble volume during condensation. Due to the two dimensional photographic technique we are not sure whether this is a real oscillation or whether due to rotating movement of the not real spherical bubble there is only the impression of an oscillation. Therefore calculating the velocity and the acceleration we took smoothed values. The figure shows that the growing velocity can amount up to 2,5 m/s and that the velocity of the phase boundary during condensation reaches values of -0,2 m/s. From this one can derivate a very high acceleration several times changing in sign

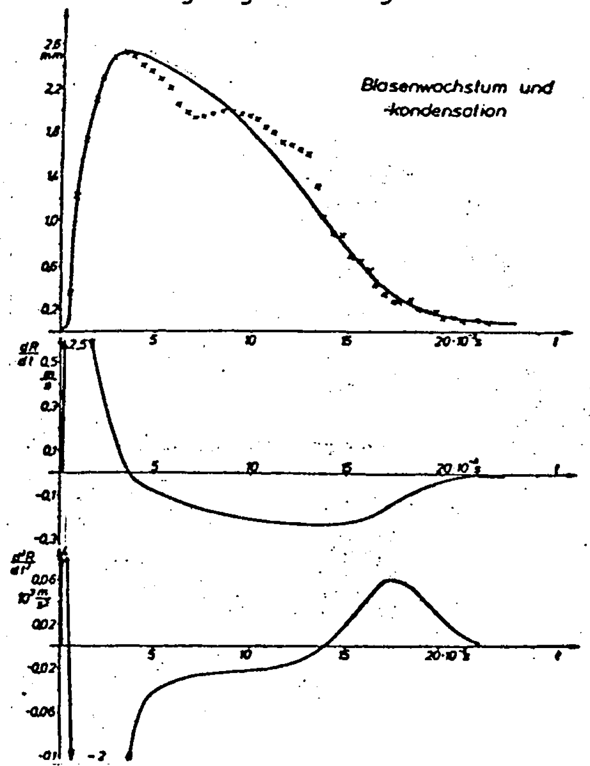

Fig. 4: Bubble growth and condensation in subcooled pool boiling with first and second derivation

The acceleration in this example was measured with 4.10^3 m/sec^2 as maximal value.

All these highly dynamic motions have to be taken into account working out a theoretical model for predicting bubble growth and collapse, and one therefore cannot limit the deliberations to the heat and mass transfer at the phase boundary. Due to the high phase velocities, usually one cannot assume thermodynamic equilibrium between vapor and liquid. Also a model, which is only based on inertia effects, does not meet the reality. An answer to the temperature conditions in the boundary and around the bubble is given by the holographic interferometry. But just these measurements show as one can see from the next two figures that the temperature distribution is formed by a process of very statistical nature. Figure 5 shows the temperature round a - what we call - "primary bubble". The black lines represent in a first approximation isotherms and one can see that there is a great temperature gradient at the bottom or root of the bubble. At its top the bubble is covered with a thin liquid layer of saturation temperature. From this it is evident that there is a great difference of surface tension forces around the bubble which may result in a eddie-circulation, improving the heat transfer process. To give an idea of the dimensions one should mention that the heater - to be seen as a black line - was 0,4 mm thick and that the bubble had a diameter of a few millimeters.

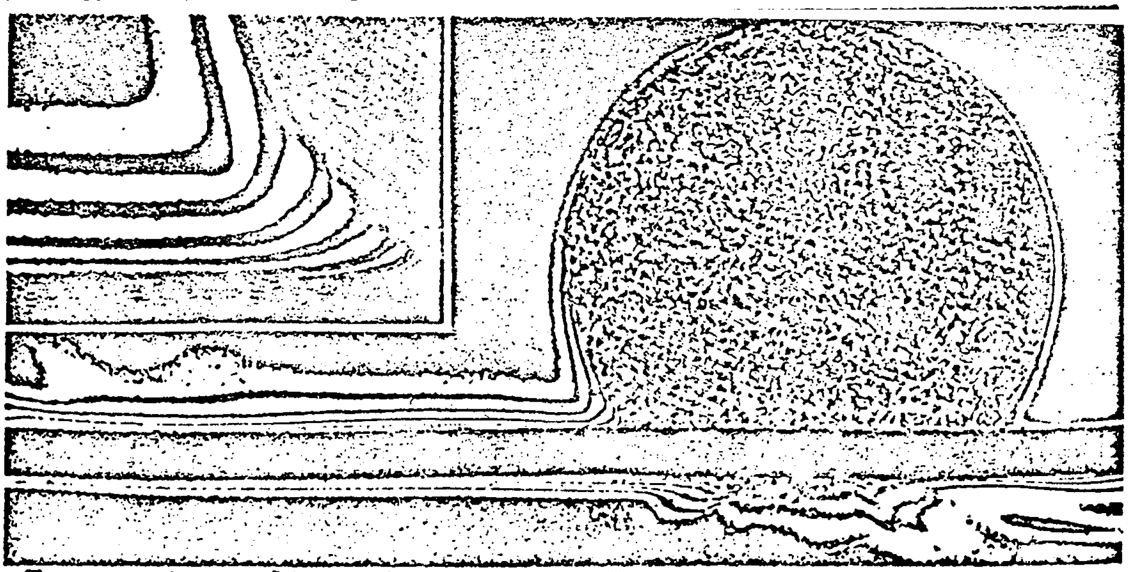

Fig. 5: Temperature distribution round a growing "primary bubble". Water, P = 0,4 bar, $\dot{q}$ = 46 W/cm^2, Subcooling 3 K

Above the top of the bubble there seems to be a uniform undisturbed temperature. This is not always the case and in figure 6 one can see quite different conditions in the phase boundary round the bubble. Here due to the drift flow of the preceeding bubble there is still a very clearly developed temperature field. The bubble growing beneath this field - we call a "secondary bubble". One can easily imagine that the growing and condensating conditions for this bubble are quite different from the one discussed before.

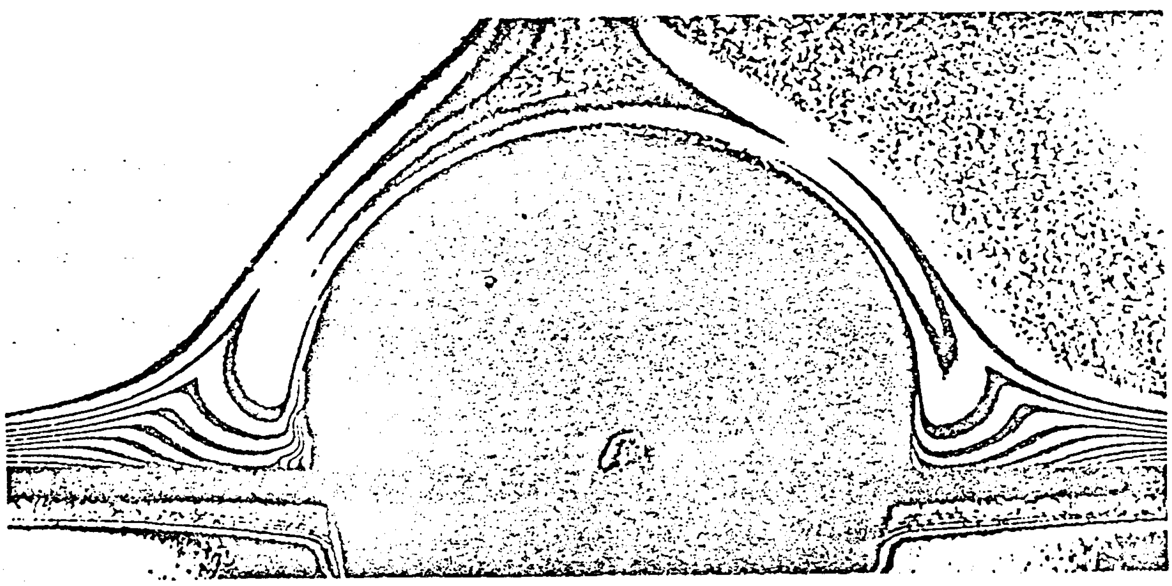

Fig. 6: Temperature distribution round a "secondary bubble". Water, p = 0,3 bar, $\dot{q}$ = 30 W/cm^2, Subcooling = 2 K

The bubbles shown in the last two figures were still in the growing period. The beginning of the condensation shows figure 7. At the top of the bubble and at its right side condensation starts as one can see from the narrowly spaced and sharply bended fringes in the phase boundary. The starting point of the recondensation usually is to be found at the top of the bubble but statistically there also may be in addition a first recondensation at any place of the upper part of the bubble. If we would follow the condensation process furtheron, we could see that after the beginning of the condensation at the top of the bubble, a very violent circulation starts round the bubble, which transports cold liquid to the bubble root and now initiates further condensation at the bubble foot whilst the top is covered with a thin layer of saturated liquid insulating the vapor from the subcooled fluid. This insulating layer is stabilized during the bubble detachment, so one could imagine to calculate the heat transfer process between vapor and liquid according to the flow conditions around a sphere with a maximum of heat transport in the eddies of the drift flow. But, in addition, there are inertia effects.

A further information for elaborating theoretical models could be taken from the temperature profile in the liquid near the wall as shown in figure 8. In this figure with the wall distance as abscissa the measured minimum and maximum temperatures are plotted. In addition there is shown the temporal mean value of the local boundary temperature. The local temperature in the boundary is subject to very strong, high frequency changes as also shown in this figure. The mean temperature was calculated by integrating this oscillating curve. In the immediate neighbourhood of the heated wall the amplitudes of the temperature changes are naturally determined by the heat capacity of the wall material. In this example the heated wall was made of a thick piece of copper having high heat capacity and good heat conductivity. The distance where the temperature oscillations reach their maximum value depends on the subcooling of the liquid and on the heat flux density. With diminishing subcooling it approaches to the wall. Under the conditions of subcooled boiling the maximum

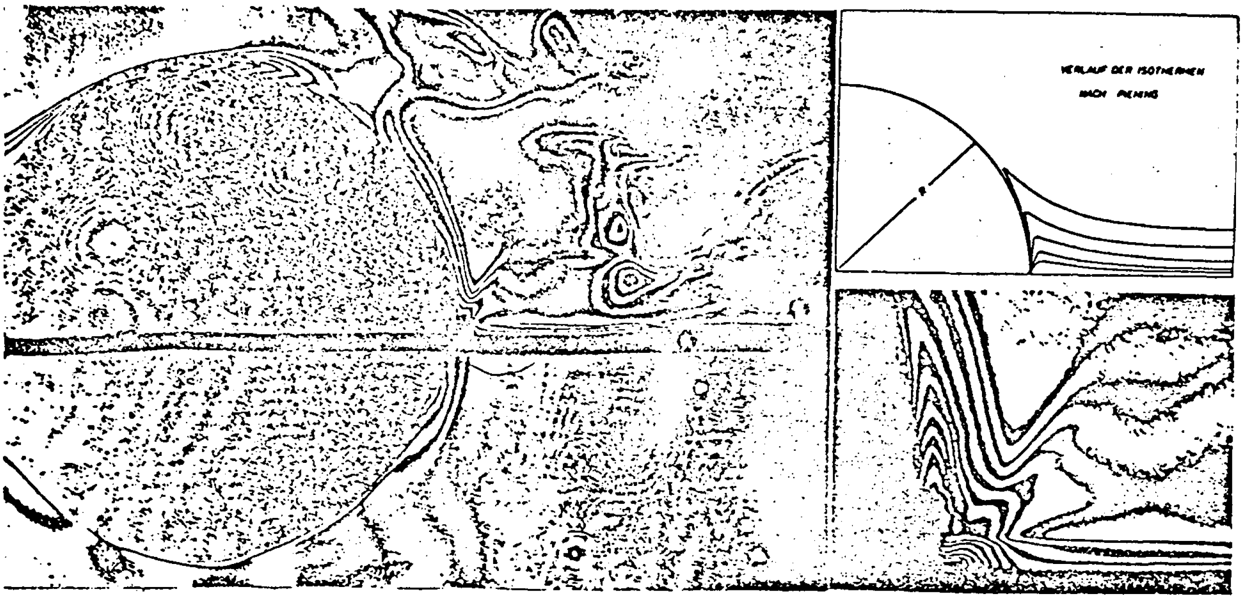

Fig. 7: Temperature distribution round a growing "primary bubble", Condensation at the top of the bubble
Water, p = 0,4 bar, $\dot{q}$ = 46 W/cm^2, Subcooling = 3 K

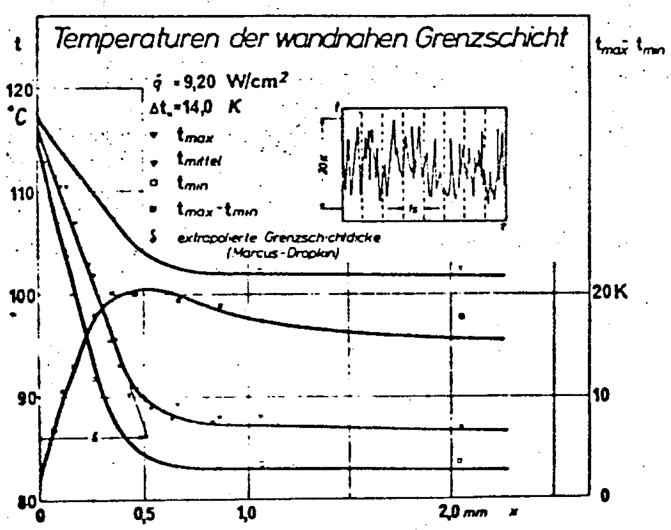

Fig. 8: Temperature profiles in the boundary layer over a heated horizontal copper plate

of these oscillations is to be expected in such a distance where the condensation starts. This is usually in a distance of 0,2 ÷ 0,5 mm. The temperature curve in the boundary layer is as the figure shows at the beginning linear and then approaches asymptotic in an e-function to the bulk temperature of the fluid.

From the literature it is known that with saturation boiling the gradient of the temperature distribution in the boundary layer becomes greater with growing heat flux density, which in a simple manner can perhaps be explained by the fact, that in the very first time till the bubble is formed, the heat transport has to be managed by pure conduction in the fluid. For calculation with subcooled boiling one also has to know the dependency of this temperature distribution on the subcooling. Figure 9 shows the time averaged curves of the

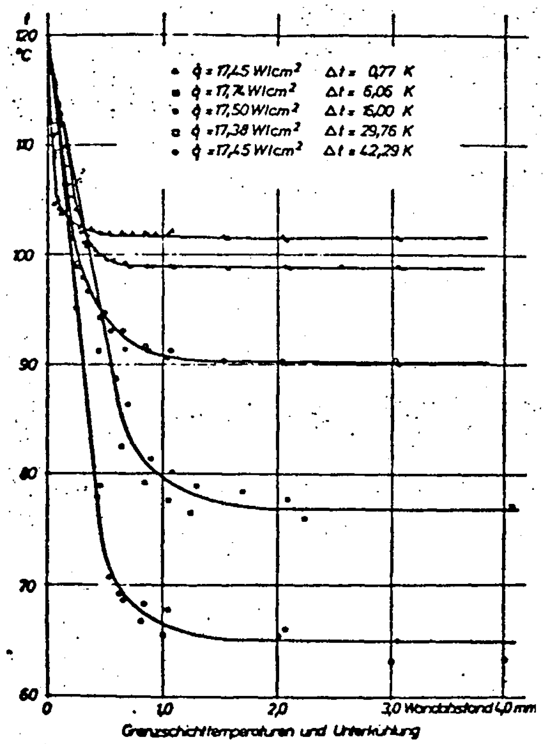

Fig. 9: Time averaged temperature profiles over a heated copper plate with various subcoolings

fluid temperature for different subcooling at constant heat flux density. One can see, that the temperature gradient in the immediate vicinity of the wall is low or only a weak function of the bulk temperature, that is of the subcooling.

From these measurements one can derive heat transfer coefficients as done in figure 10 for low pressure free convection subcooled boiling. The heat transfer behaviour shown there reaches from values - measured at low heat flux densities - which fully correspond to pure free convection conditions up to the high values of fully developed saturation boiling.

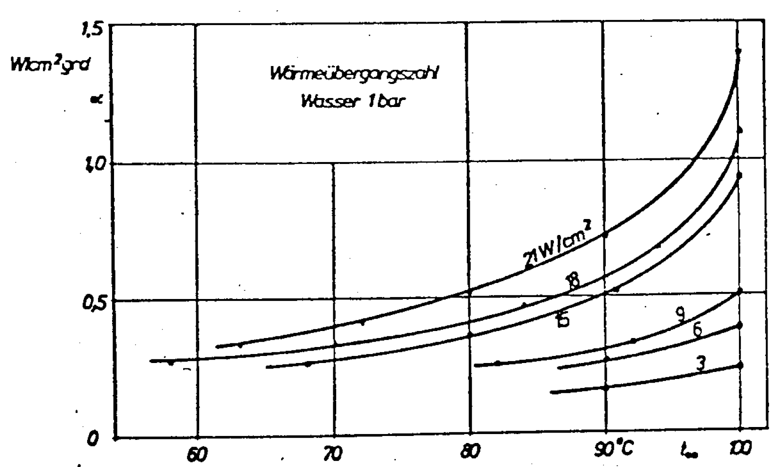

Fig. 10: Heat transfer coefficients with subcooled boiling at free convection. Water, p = 1 bar

3. Measurements of bubble formation at forced convection

The measurements presented up to now were all taken with free convection and low pressures. From additional measurements under forced convection we got the impression that the conditions for the bubble formation in the boundary layer near to the wall don't change appreciably. This may be due to the fact that the bubbles at the wall break down the flow velocity very much so that the axial convection has no or little influence on the bubble formation. This is well known for the behaviour of the heat transfer coefficient with boiling. There is - up to a certain quality - almost no influence of flow velocity. But the mass flow rate influences via the energy balance the bulk temperature and thus the subcooling of the liquid. In a technical heat exchanger, in addition, the complicated geometrical conditions, for example the arrangement of the rods in a cluster, have an influence on the first bubble formation as well as on bubble growth and collapse.

A very important question for calculating subcooled boiling is the determination of the location where under forced condition in a channel the first bubble formation starts. Measurements done at high pressure and at flow rates in the technical interesting range, showed that the beginning of bubble formation is also strongly dependent on the shape of the cross section of the channel. In figure 11 experimental

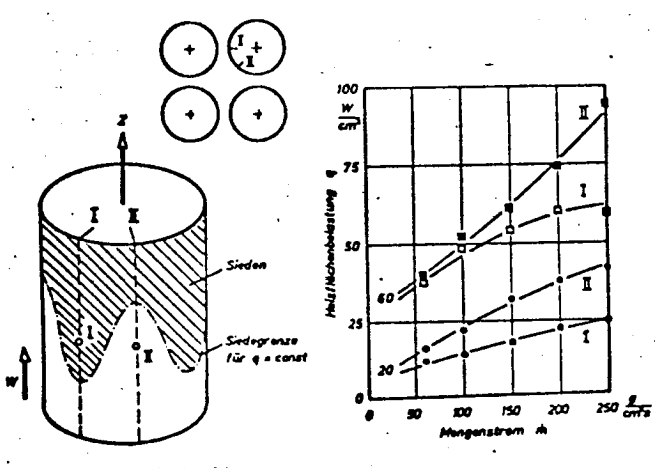

Fig. 11: Beginning of bubble formation in a four-rod bundle at

results are presented which were gained in a four-rod bundle. The rods of this bundle were uniformly heated. It is easy to imagine that in the most narrow gap between two rods there is a smaller mean velocity than in the diagonal position. Therefore boiling first starts in the narrow gap and the boundary line for incipient boiling has as shown in the figure a cosine form around the circumference of each rod. The heat flux conditions under which subcooled boiling at these circumferencial rod positions starts may differ very strongly as shown in the right part of this figure.

In figure 12 for a pressure of 100 bar and for different mass flow rate densities the beginning of subcooled boiling is plotted. One can see that at low subcooling boiling already starts at very small heat flux densities and even at high subcooling the heat flux conditions for incipient boiling lay well within the range given sometimes in oil-or gas-fired boilers but always present in pressurized water reactors.

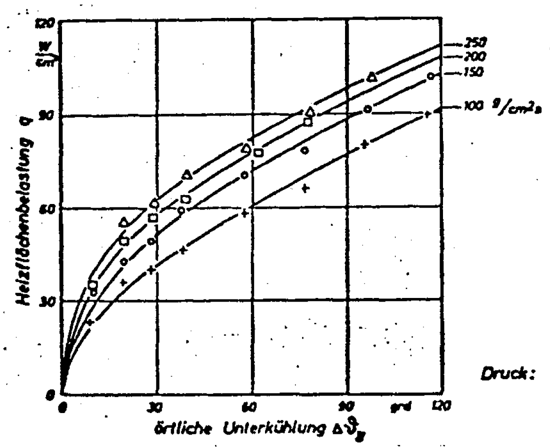

Fig. 12: Beginning of subcooled flow boiling in a four-rod bundle

But these basic considerations of the physical process would not be complete, if we would not mention very shortly another in this connection very important typical behaviour of two phase flow, before treating theoretical models for calculating void fraction in subcooled boiling. Thus in addition we have to pay attention to the slip ratio which, as pointed out already in detail in this curse, is a function of different hydrodynamic and thermodynamic parameters like mass flow rate, pressure and quality.

4. Theoretical models

For calculating the void fraction in subcooled boiling we have to know

1. the location or the subcooled temperature in the channel where the first bubbles are formed at the heated wall
2. the subcooled temperature or the location in the channel where the vapor bubbles detach from the wall and
3. the heat and energy distribution in the fluid, i.e. the amount of heat which got into the liquid and into the vapour respectively. One has to keep in mind, that there is thermodynamic non-equilibrium, i.e. the mean specific enthalpy of the fluid is smaller than the enthalpy of the saturated liquid
4. the recondensation rate of the detached bubbles in the channel from the point where they departed from the wall up to the location where saturated boiling is reached
5. the slip ratio.

From the several theoretical models /1/, /2/, /3/, /5/ for predicting void fraction in subcooled boiling, we shall shortly discuss the basic ideas of Bowring /1/ and than we shall treat the model of Rouhani /5/ more in detail.

4.1 Bowring's model

In figure 1 we discussed qualitatively the amount of void fraction at subcooled boiling. Figure 13 shows a more detailed sketch of the heat transfer domains in a heated channel.

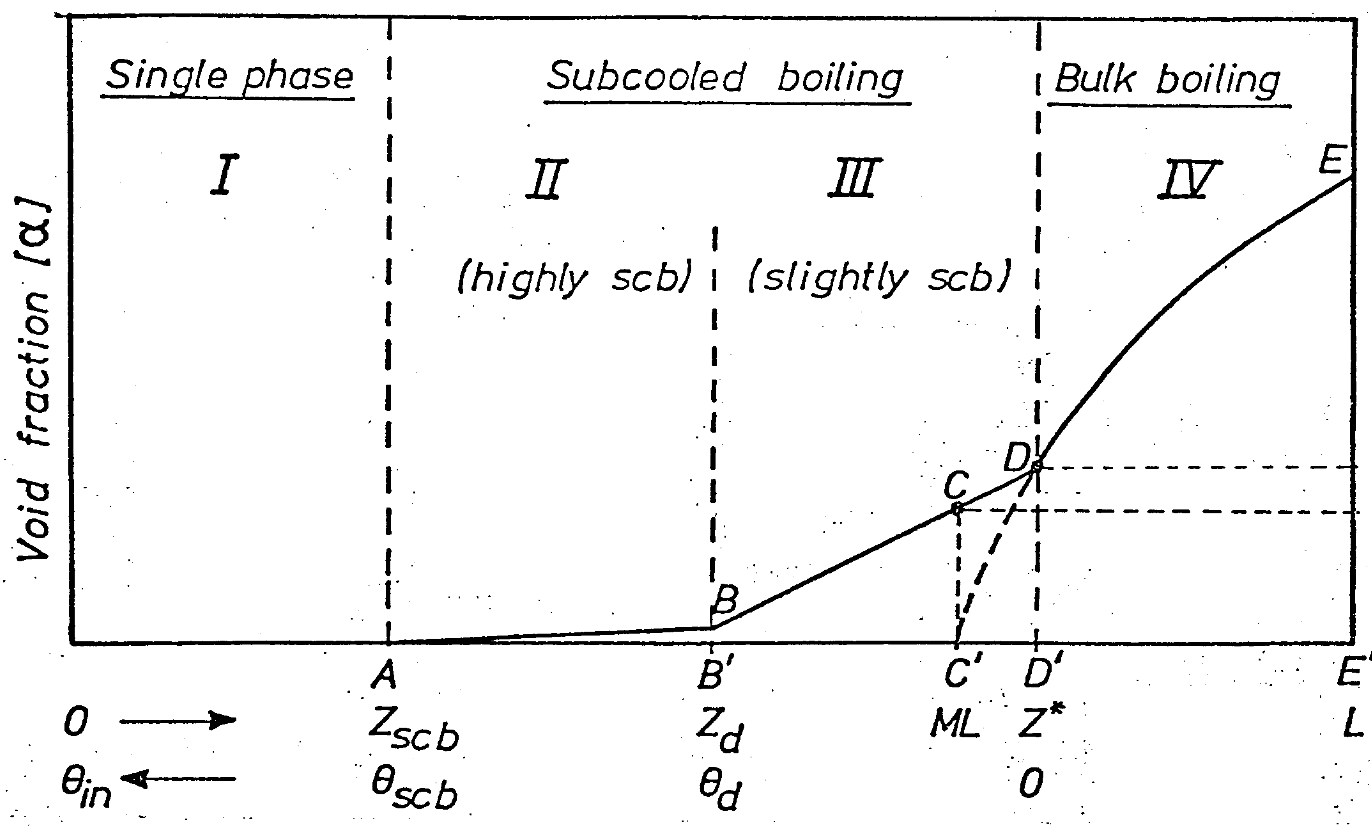

Fig. 13: Sketch of the heat transfer domains in a heated channel

Bowring assumes that the first area (AB in fig.13) has almost no contribution to the steam voidage in the channel and that a rapid increase in void fraction starts from the point B in fig. 13 which represents the condition for bubbles to leave the surface. It is this second region BCD which is the main subject of his theory.

For a long heated channel with heat transfer regions as shown in fig. 13 the transition sub-cooling θ_r at which the void fraction begins to rise rapidly is that, at which the bubble detachment occurs. This is not, however, a unique criterion for θ_r, as the sub-cooling at which boiling starts θ_{scb} may be lower than the temperature difference at which the bubbles detach (θ_d). A third possibility is that the channel inlet sub-cooling may be less than both above-mentioned temperatures that is less than θ_{scb} and less than the detachment subcooling θ_d.

We now need a condition for sub-cooled boiling to start which is simply defined by Bowring as that condition when the heated surface temperature reaches saturation temperature plus the degree of superheat given by an equation for nucleate boiling heat transfer for example by the equation of Jens and Lottes /11/.

$$t_w - t_s = \beta \cdot (q/A)^{0,25} \qquad (1)$$

By doing so we get for the temperature at which boiling starts

$$\theta_{scb} = q/(A \cdot h) - \beta \cdot (q/A)^{0,25} \qquad (2)$$

In this equation β has the value $7{,}8 \cdot \exp[-0{,}0163(p-1)]$. The units used in this equation are K, W, m, atmospheres, respectively. One also can - as done by Bowring - evaluate the sub-cooling at which detachment occurs in a pure empirical way with the help of an empirical parameter. From measurements Bowring assumes that the temperature at which detachment occurs is a function of velocity, heat flux and pressure but not of geometry. This is only true - as shown in chapter 3 - if the local velocity is known. Bowring correlates the temperature at which the bubbles detach by the simple equation

$$\theta_d = \eta \cdot q/(A \cdot v) \qquad (3)$$

in which η is a so-called subcooled void parameter only depending on the pressure of the liquid. For water Bowring gives the relation

$$\eta = \frac{v \cdot \Theta_d}{q/A} = (14 + 0{,}1 \cdot p) \qquad (4)$$

This equation is valid in the range between 11 and 136 atmospheres. The units in these equations are again W, K, m, s and atmospheres.

Heat is removed from the heated surface during boiling by several simultaneous mechanisms:

1. as latent heat contempt of bubbles $(q/A)_e$
2. by convection caused by bubble agitation of the boundary layer $(q/A)_a$
3. by condensation at the top of the growing bubble while still attached to the wall
4. by single phase heat transfer between patches of bubbles $(q/A)_{sp}$.

The first three mechanisms are discussed by Forster and Greif /12/ who showed that the second i.e. $(q/A)_a$ was sufficient to account for the high heat transfer at atmospheric pressure without introducing the third. The evaporative heat flux $(q/A)_e$ is small at atmospheric pressure but increases with pressure.

Neglecting the third mechanism we may write

$$(q/A) = (q/A)_e + (q/A)_a + (q/A)_{sp} \qquad (5)$$

It may readily be shown that

$$(q/A)_e = n \cdot f \cdot B_d \cdot \rho_g \cdot \lambda \qquad (6)$$

The agitation heat flux $(q/A)_a$ arises in the following manner. As a bubble grows, it pushes superheated liquid in the front of it into the sub-cooled bulk fluid. When it detaches from the heated surface, it continues to push hot liquid away from the wall in the drift flow, while colder liquid from the bulk rushes to the wall. Thus a quantity of cold water related to the bubble volume is drawn to the heated surface heated through an effective temperature difference τ and pushed out again by the bubble. Thus we may write

$$(q/A)_a = n \cdot f \cdot B_d \cdot \rho_l \cdot C_p \cdot \tau \tag{7}$$

where τ is related to the bulk temperature, the degree of superheat at the wall and the efficiency with which the bubble circulates the water. Therefore Bowring has to introduce a second empirical parameter π defined as

$$\pi = \frac{(q/A)_a}{(q/A)_e} = \frac{\rho_l \cdot C_p}{\rho_g \cdot \lambda} \tag{8}$$

whose value must be obtained from experimental data.

Bowring furthermore assumes that the heat flux in subcooled boiling at a given surface temperature is independent of subcooling and velocity, if boiling is "fully developed". Finally Bowring assumes as already mentioned, that the heat exchange due to bubble recondensation is neglectable, this consequently means that bubble collapse is so small that it has no influence on the void. This certainly is only true in the slightly subcooled region.

Using the mass balance equations for both the subcooled and the bulk boiling region, Bowring gets the following relationship between void fraction and mass fraction quality.

$$\alpha/(1-\alpha) = (\chi/S) \cdot \frac{\rho_l}{\rho_g} \cdot (1-\chi) \tag{9}$$

In the bulk boiling region all the heat transfer is eventually converted to latent heat of evaporization so that the steam quality can be written as

$$\chi = [(P_h/\rho_l)\cdot A\cdot v\cdot\lambda] \cdot \int_{ML}^{z} (q/A)\, dz \quad ; \; z > z^* \; . \qquad (10)$$

in the slightly subcooled region only the evaporative component of the heat flux $(q/A)_e$ is used to produce steam, the rest going to raise the bulk temperature. The equation describing the rise of steam quality may be written analogous with the bulk boiling equation as

$$\chi_b = [(P_h/\rho_l)\cdot A\cdot v\cdot\lambda] \cdot \int_{z_r}^{z} (q/A)_e\, dz \qquad (11)$$

Substituting from equations (5) and (8) where θ_{sp} is given by

$$(q/A)_{sp} = h\cdot\theta \quad ; \quad \theta > \theta_{sp} \qquad (12)$$

where

$$\theta_{sp} = 0{,}7\cdot q/(A\cdot h) - \beta\cdot(0{,}7\cdot q/A)^{0{,}25} \qquad (13)$$

In terms of void fraction one finally gets

$$\frac{\alpha_b\cdot(1-\chi_b)}{1-\alpha_b} = \frac{P_h}{\rho_g\cdot A\cdot v\cdot\lambda\cdot S}\cdot\int_{z_r}^{z}\frac{(q/A)-(q/A)_{sp}}{(1+\pi)}\, dz \qquad (14)$$

or integrated

$$\frac{\alpha_b\cdot(1-\chi_b)}{1-\alpha_b} = \frac{P_h}{\rho_g\cdot A\cdot v\cdot\lambda\cdot S(1+\pi)}\cdot(\overline{q/A})_{scb}\cdot(z-z_r) \qquad (15)$$

In this equation z_r is the point in the channel where θ_r is reached, which can be easily calculated if the longitudinal heat flux distribution is known and if no mixing is assumed.

The weight fraction X_b may be calculated from

$$X_b = \frac{P_h}{\rho_l \cdot A \cdot v \cdot \lambda (1+\pi)} \cdot \overline{(q/A)}_{scb} \cdot (z - z_r) \qquad (16)$$

In most cases X_b is small and may be neglected.

4.2 Theory given by Rouhani

A detailed literature survey on the papers dealing with the calculation of void fraction in subcooled boiling was given by Rouhani /5/. Several additional reports on this subject have been appeared in the literature recently. Among these are the works of Levy /2/ and Zuber /13/. The main points of the paper by Levy /2/ is a new method of calculating the liquid subcooling at the point of the bubble departure. This is different from Bowrings' method /1/. Levy suggests also a certain relationship between the true local vapor void fraction and the corresponding thermal equilibrium value. Finally by applying an accepted slip correlation he calculates the void fraction in subcooled boiling.

Rouhani considers two regions of subcooled boiling:

1. Local boiling with bubbles not detaching from the heated surface and high subcooling,
2. local boiling with bubble detachment and flow of vapor bubbles with liquid.

He assumes that the maximum value of wall voidage α_c occurs at the end of the first region. He calculates the voidage at this point by the simple equation

$$\alpha_c = 2{,}435 \cdot 10^{-3} \cdot p^{-0{,}237} \cdot \frac{P_h}{A_c} \qquad (17)$$

This equation is not written in a dimensionless form and therefore the pressure p is to be given in N/m^2 and it is only valid for water as boiling medium. P_h is the perimeter and A_c is the flow area in the channel with the dimension m and m^2 respectively.

Rouhani assumes that local boiling starts at a point where

$$\frac{q}{A} - h\,(t_s - t_l) > 0 \qquad (18)$$

In this equation h is the single phase heat transfer coefficient for only liquid flow which can be calculated by using Collburn's correlation

$$h = \frac{0{,}023}{Re^{0{,}8}} \cdot G \cdot C_p \cdot Pr^{0{,}8} \qquad (19)$$

At high subcooling the single phase heat transfer will be still effective but accompanied by the other mechanisms. With decreasing subcooling the heated surface will become more and more covered with bubbles and hence less accessible to the bulk liquid flow. For this reason Rouhani assumes that the non-boiling fraction of the heat flux will be

$$\left(\frac{q}{A}\right)_{nb} = \left(1 - \frac{\alpha}{\alpha_c}\right) \cdot h \cdot (t_s - t_l) \quad ; \quad \alpha > \alpha_c \qquad (20)$$

in which α is the local void fraction and α_c is the void fraction at the point of vapour clotting.

The total heat balance on the heated surface is

$$\frac{q}{A} = h \cdot \Theta_l \cdot \left(1 - \frac{\alpha}{\alpha_c}\right) + \dot{m}_s \cdot \lambda + \frac{\dot{m}_s}{\rho_g} \cdot C_p \cdot \rho_l\, \Theta_l \qquad (21)$$

the amount of the heat which goes to the steam generation can easily be calculated by a simple energy balance.

$$dQ_b = \dot{m}_s \cdot \lambda \cdot P_h \cdot dz = \frac{(q/A) - h \cdot \Theta_l [1 - (\alpha/\alpha_c)]}{\rho_g \cdot \lambda + C_p \cdot \rho_l \cdot \Theta_l} \qquad (22)$$

Finally the amount of heat which goes to the subcooled liquid through condensation of vapor bubbles can be expressed as

$$dQ_c = k_c \cdot \Theta_l \cdot dz \qquad (23)$$

in which k_c is a condensation coefficient with the same dimensions as that of the thermal conductivity. Rouhani shows that this constant is actually proportional to the thermal conductivity of the liquid phase divided by the Prandtl-number.

Considering the heat balance in the axial direction one can use two separate equations for each of the two phases. Both phases are connected by equation (23), which gives the heat transfered from the vapor to the liquid.

For the vapor one can generally write the heat balance as

$$dx = \frac{dQ_b - dQ_c}{\dot{m} \cdot \lambda} \qquad (24)$$

This is the differential change in the true vapor fraction with dz regardless of the flow regime or slip ratio.

The total heat balance across dz gives the differential change in the true liquid subcooling as

$$d\theta_l = \frac{(q/A) \cdot P_h \cdot dz - (dQ_b - dQ_c)}{\dot{m} \cdot C_p \cdot (1 - x)} \qquad (25)$$

Now assuming that the variations of dQ_b and dQ_c with z are known, one may integrate equation (24) to obtain the true steam quality at any height. In addition one has to use a suitable relationship for slip ratio and can calculate the local values of the void volume fraction.

For the relationship between the local void and the mass flow rate quality, that is for indirectly expressing the slip ratio, Rouhani used a derivation given by Zuber and Findlay /14/ which has the form

$$\alpha = \frac{x}{\rho_g} \left\{ C \left[\frac{x}{\rho_g} + \frac{1-x}{\rho_l} \right] + \frac{1{,}18}{G} \cdot \left[\frac{\sigma \cdot g(\rho_l - \rho_g)}{\rho_l^2} \right]^{0{,}25} \right\}^{-1} \qquad (26$$

In this equation, C is a distribution parameter which is dependent on the velocity profile and void distribution over flow area. Rouhani found that over a wide field of parameters an average value of about 1.1 for this distribution parameter C is adequate and only at lower mass velocities a strong dependence of C on this parameter is given, and in this region the value may be much larger than 1.1.

The condensation coefficient k_c is dependent on many parameters, for example on the thermal conductivity of the liquid, on the local values of contact area between vapor and liquid, which depends on the void fraction, on channel geometrie, on mass velocity and on heat flux density. In a systematic analysis comparing experimental data with analytical deliberations Rouhani found the following correlation for the condensation coefficient k_c.

$$k_c = a \cdot \frac{k_l}{Pr} \cdot \left(\frac{\rho_g}{\rho_l}\right)^2 \cdot A_c^{2/3} \cdot \alpha^{2/3} \cdot Re_l / N_q^{0.5} \qquad (27)$$

In this equation k_l is the thermal conductivity of the liquid, Pr the Prandtl-number, Re the Reynolds-number and Nq a dimensionless number taking in account the effect of heat flux on the heat transport between vapor and liquid as given in equation (28).

$$N_q = \frac{(q/A) \cdot \mu_l}{\lambda \cdot (\rho_l - \rho_v)} \qquad (28)$$

Finally in equation (28) a is a dimensional constant, which probably may depend on the number of nucleation sites per unit area of the heated surface and on the bubbling frequency, which has an value of 30 $m^{-4/3}$.

Thus by numberical integration of the equations (22) - (25) and using equations (26, 27, 28) one can calculate the local void fraction with subcooled boiling.

Rouhani made a very detailed comparison of his theoretical results with experimental data. In fig.14

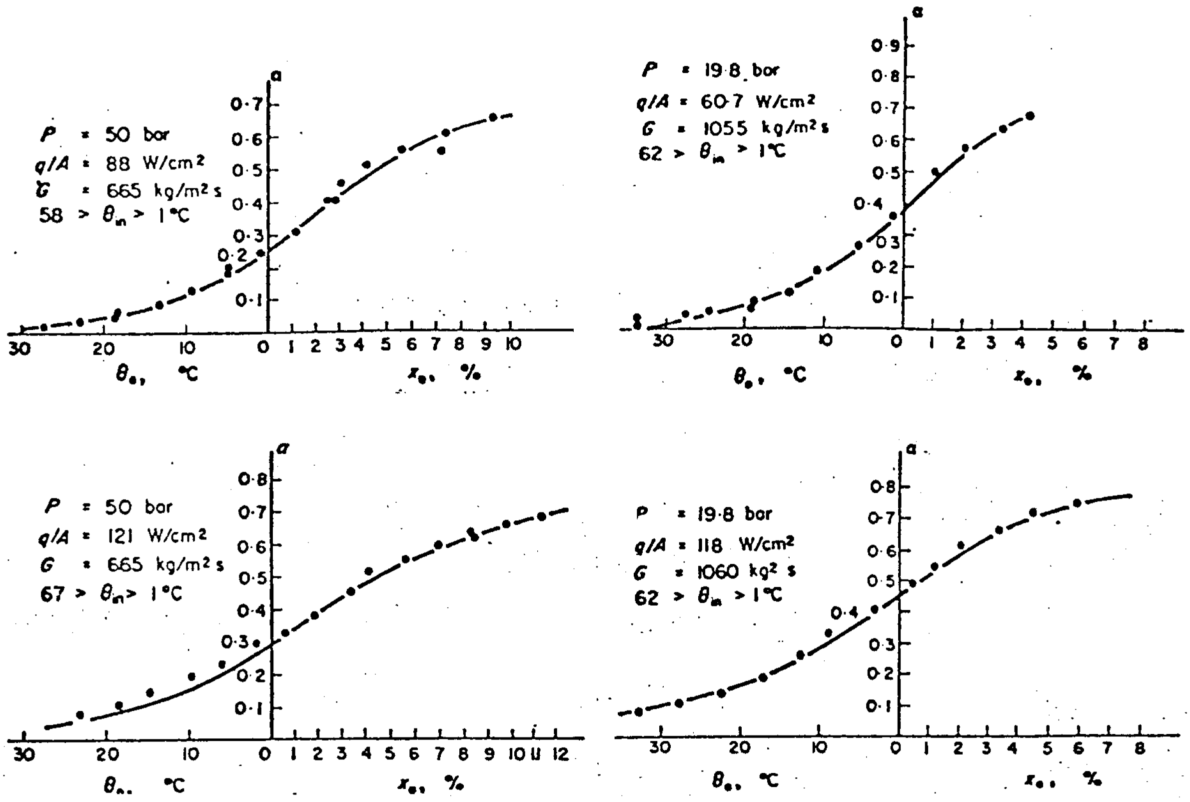

Fig. 14: Comparison of calculated data with data of Ref. [15]

an example for such a comparison is given. This example shows, that there is an excellent agreement between measurements and theory. Only at very low mass velocities, i.e. below 130 kg/m^2s the theory gives a void, which is about 20% higher than the measurements. This may be due to an effect of mass velocity on the rate of condensation or on the effect of variations of physical properties because of the large temperature variations. There certainly also may arise an intensified convection at very low mass velocities and a heat removal by such mechanisms has not been accounted for in the calculations.

Generally one can say that based on the results of comparison the model of Rouhani gives a very close approximation of the physical phenomena involved in the changes of void fraction in flow boiling throughout the boiling regions.

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θ_l	liquid subcooling as an integral of equation (25), $t_s - t_l$ [°C]
θ_d	subcooling at bubble formation
θ_o	average liquid subcooling obtained from a heat balance for the whole flow [°C]
ε	a pressure-dependent non-dimensional parameter
η_l	subcooled void parameter - value given by eq. (4)
λ	latent heat of vaporization [J/kg]
μ_l	dynamic viscosity of liquid [kg/ms]
π	subcooled void parameter - value given by eq. (8)
ρ_g	vapour density [kg/m^3]
ρ_l	liquid density [kg/m^3]
σ	surface tension of liquid [N/m]
τ	$\pi \cdot (\rho_g \cdot \lambda)/(\rho_l \cdot c_p)$

Subscripts

a	agitation
b	bubble
e	"evaporative"
d	at bubble detachment
M	at z = ML
s	saturation
scb	subcooled boiling
sp	single phase
r	at the point of rapid rise, in subcooled void fraction
w	wall
-	average value
*	at intersection of subcooled and bulk boiling void curves