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Newton's temperature scale and the law of cooling*

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Abstract. Isaac Newton (1642–1727) published his temperature scale in 1701; its significance lies both in its range of temperature and in its instrumentation. Its presentation also includes "Newton's Law of Cooling". Newton's observed values, interpreted in the light of current knowledge, generally correspond quite well with our accepted temperature values.

Newtons Temperaturskala und das Abkühlungsgesetz

Zusammenfassung. Isaac Newton (1642–1727) veröffentlichte 1701 eine Temperaturskala, die sowohl wegen ihres großen Meßbereichs als auch wegen der verwendeten Instrumente von Bedeutung ist. Zu ihrer Aufstellung wurde "Newton's Law of Cooling" definiert. Interpretiert man Newtons Meßergebnisse nach unserer heutigen Kenntnis, stimmen sie befriedigend mit den heutigen Temperaturwerten überein.

1 Introduction

Every engineer knows Newton's "Law of Cooling", the formal basis of convective heat transfer; but only few know the origin of this law and hardly anybody has read the original paper [1].

In this paper Newton establishes a temperature scale which takes as fixed-points not only the boiling-points of liquids, but also for the first time the melting- and freezing-points of metals and alloys. This scale is also remarkable for its wide range of temperature (over 600 °C). The following sections will describe Newton's work and interpret his results in the light of the current state of knowledge. There will also be references to earlier commentaries [2–4].

2 The temperature scale

As in modern temperature scales, Newton's temperature scale consists of a definition and description of physical phenomena, chosen for their utility as temperature fixed-points, and establishes numerical values for these, which Newton sets out in two columns; one in arithmetic progression (left in Fig. 1), the other in geometric progression (middle in Fig. 1). Here we will only consider the left column. Newton's paper also includes an instruction for interpolating temperatures between the fixed-points, together with a description of his measuring apparatus. Newton uses two thermometers: for the lower range, a linseed-oil-in-glass thermometer, and for the upper range, a calorimeter, from whose rate of cooling temperature is calculated. For this purpose Newton defines his "Law of Cooling".

For the following to be more readily understood, we will now introduce two new terms: the quantity "Newton temperature" (t_N) and the unit "degree Newton" (°N). Newton temperature may be understood as those temperatures deriving solely from his own experimental results. These temperatures are for him *the* real temperatures. He calibrates his thermometer not by comparison with a standard instrument, but he conducts definitive experiments. The Celsius temperatures which are today accepted to be correct, we term t. The unit "degree Newton" (°N) is given to the numerical values of the left column in Fig. 1. Newton's scale has two defining fixed-points: the ice-point, where the Newton temperature $t_N = 0\,°N$, and the temperature of the human body, where $t_N = 12\,°N$. We may expect a linear relation between the Newton scale and the Celsius scale. Any deviations between the two temperature scales are only conceivable at higher temperatures.

We therefore set:
for the ice-point

$$t_N = t = 0\,°N = 0\,°C$$

and for the body temperature

$$t_N = t = 12\,°N = 37\,°C$$

hence giving the relation

$$1\,°N = 3.08\,°C. \qquad (1)$$

* Dedicated to Professor Eckert on the occasion of his 80th birthday

Scala graduum Caloris.

Calorum Deſcriptiones & ſigna.

0		CAlor aeris hyberni ubi aqua incipit gelu rigeſcere. Innoteſcit hic calor accurate locando Thermometrum in nive compreſſa quo tempore gelu ſolvitur.
0,1,2,		Calores aeris hyberni.
2,3,4.		Calores aeris verni & autumnalis.
4,5,6.		Calores aeris æſtivi.
6		Calor aeris meridiani circa menſem Julium.
12	1	Calor maximus quem Thermometer ad contactum corporis humani concipit. Idem circiter eſt calor avis ova incubantis.
14 3/11	1 1/4	Calor balnei prope maximus quem quis manu immerſa & conſtanter agitata diutius perferre poteſt. Idem fere eſt calor ſanguinis recens effuſi.
17	1 1/2	Calor balnei maximus quem quis manu immerſa & immobili manente diutius perferre poteſt.
20 2/11	1 3/4	Calor balnei quo cera innatans & liquefacta defervendo regiſcit & diaphaneitatem amittit.
24	2	Calor balnei quo cera innatans incaleſcendo, liqueſcit & in continuo fluxu ſine ebullitione conſervatur.
28 6/11	2 1/4	Calor mediocris inter calores quo cera liqueſcit & aqua ebullit.
34	2 1/2	Calor quo aqua vehementer ebullit & miſtura duarum partium plumbi trium partium ſtanni & quinque partium biſmuti defervendo rigeſcit. Incipit aqua ebullire calore partium 33 & calorem partium pluſquam 34 1/2 ebulliendo vix concipit. Ferrum verò deferveſcens calore partium 35 vel 36, ubi aqua calida & 37 ubi frigida in ipſum guttatim incidit, deſinit ebullitionem excitare.
40 4/11	2 3/4	Calor minimus quo miſtura unius partis Plumbi quatuor partium Stanni & quinque partium Biſmuti incaleſcendo liqueſcit, & in continuo fluxu conſervatur.
48	3	Calor minimus quo miſtura æqualium partium ſtanni & biſmuti liqueſcit. Hæc miſtura calore partium 47 defervendo coagulatur.
57	3 1/4	Calor quo miſtura duarum partium ſtanni & unius partis biſmuti funditur, ut & miſtura trium partium ſtanni & duarum plumbi ſed miſtura quinq; partium ſtanni & duarum partium biſmuti hoc calore defervendo rigeſcit. Et idem facit miſtura æqualium partium plumbi & biſmuti.
68	3 1/2	Calor minimus quo miſtura unius partis biſmuti & octo partium ſtanni funditur. Stannum per ſe funditur calore partium 72 & Defervendo rigeſcit calore partium 70.
81	3 3/4	Calor quo biſmutum funditur ut & miſtura quatuor partium plumbi & unius partis ſtanni. Sed miſtura quinque partium plumbi & unius partis ſtanni ubi fuſa eſt & defervet in hoc calore rigeſcit.
96	4	Calor minimus quo plumbum funditur. Plumbum incaleſcendo funditur calore partium 96 vel 97 & defervendo rigeſcit calore partium 95.
114	4 1/4	Calor quo corpora ignita defervendo penitus deſinunt in tenebris nocturnis lucere, & viciſſim incaleſcendo incipiunt in iiſdem tenebris lucere ſed luce tenuiſſima quæ ſentiri vix poſſit. Hoc calore liqueſcit miſtura æqualium partium Stanni & Reguli martis, & miſtura ſeptem partium biſmuti & quatuor partium ejuſdem Reguli defervendo rigeſcit.
136	4 1/2	Calor quo corpora ignita in tenebris nocturnis candent, in crepuſculo vero neutiquam. Hoc calore tum miſtura duarum partium reguli martis & unius partis Biſmuti tum etiam miſtura quinq; partium reguli martis & unius partis Stanni defervendo rigeſcit. Regulus per ſe rigeſcit calore partium 146.
161	4 3/4	Calor quo corpora ignita in crepuſculo proxime ante ortum ſolis vel poſt occaſum ejus manifeſto candent in clara vero diei luce neutiquam, aut non niſi perobſcure.
192	5	Calor prunarum in igne parvo culinari ex carbonibus foſſilibus bituminoſis conſtructo & abſq; uſu follium ardente. Idem eſt calor ferri in tali igne quantum poteſt candentis. Ignis parvi culinaris qui ex lignis conſtat calor paulo major eſt nempe partium 200 vel 210. Et ignis magni major adhuc eſt calor, præſertim ſi follibus cieatur.

Nnnnn 2

Fig. 1. Newton's temperature scale in the original published version [1]

A further fixed-point is the boiling-point of water, where $t_N = 33\,°N$. There are a number of fixed-points established by Newton which are not easily reproducible. However, in addition, Newton used 18 fixed-points given by the melting- and freezing-points of various alloys of the metals lead (Pb), tin (Sn), bismuth (Bi) and antimony (Sb). The latter is referred to by Newton under its alchemical name "Regulus Martis" [5]. These reproducible fixed-points are shown in Table 1, together with the proportions of the four metals by mass, the Newton temperature t_N in °C and their Celsius temperatures t, also in °C. These Celsius temperatures t are extracted from melting diagrams to be found for all existing alloys in the volumes of "Gmelin" [6]. These fixed-points, lettered $A - V$, are set out in order of increasing temperatures.

The discrepancies between the Newton and Celsius temperatures may be readily observed in Table 1, and these will be considered in detail in the following sections.

Table 1. Reproducible fixed-points from Newton's temperature scale (M Melting-point, F Freezing-point)

	M F	Pb	Sn	Bi	Sb	t_N °C	t °C
A		Ice-point				0	0
B		body temperature				37	37
C		steam-point				101,6	100
D	F	2	3	5		105	105
E	M	1	4	5		124	135
F	F		1	1		145	151
	M		1	1		148	151
G	M		2	1		176	184
H	M	2	3			176	187
I	F		5	2		176	195
K	F	1		1		176	153
L	M		8	1		209	218
M	F		1			216	232
	M		1			219	232
N	M			1		249	271
O	M	4	1			249	273
P	F	5	1			249	282
Q	F	1				293	327
	M	1				296	327
R	M		1		1	351	420
S	F			7	4	351	469
T	F			1	2	419	561
U	F		1		5	419	587
V	F				1	450	631

3 The linseed-oil thermometer

For that part of the scale between the ice-point and the tin-point (232 °C) Newton used a linseed-oil-in-glass thermometer, which we may imagine to resemble the then-current "Florence thermometer": a glass bulb with an attached capillary sealed at the top. Newton set the volume of linseed-oil as proportional to temperature; and to the volume of oil at the ice-point V_0 he gave the arbitrary value 10 000. At the body temperature $t_N = 12\,°\mathrm{N}$ he measured $V_{12} = 10256$. Therefore he established that for any Newton temperature t_N with the observed oil volume V_N the following equation must hold:

$$t_N = \frac{V_N - V_0}{V_{12} - V_0} \cdot 12\,°\mathrm{N} = \frac{V_N - V_0}{V_{37} - V_0} \cdot 37\,°\mathrm{C}. \qquad (2)$$

This is the interpolation formula for the range of the linseed-oil thermometer. If we define our standard form of the expansion coefficient β_N, then we have

$$\beta_N = \frac{V_N - V_0}{V_0 \cdot t_N} = \frac{V_{12} - V_0}{V_0 \cdot 12\,°\mathrm{N}} = 2133 \cdot 10^{-6}\,(°\mathrm{N})^{-1}$$
$$= 692 \cdot 10^{-6}\,\mathrm{K}^{-1}. \qquad (3)$$

From this we may express the Newton temperature with the following equation:

$$t_N = \frac{V_N - V_0}{V_0 \cdot \beta_N}. \qquad (4)$$

These interpretations would hold true if the expansion coefficient β were constant, that is, independent of temperature, and also if the thermometer were fully immersed, so that no correction for the exposed thread were necessary. However, neither is the case.

The mass of the linseed-oil (m) can be expressed as the product of the volume V_0 at the ice-point temperature t_0 and the density ϱ_0 at the same temperature. It may also be expressed as the sum of the product of the immersed volume V_E and the density ϱ_t at the real temperature t and the product of the thread volume $(V_N - V_E)$ and the density ϱ_F at the thread temperature t_F:

$$m = V_0\,\varrho_0 = V_E\,\varrho_t + (V_N - V_E)\,\varrho_F. \qquad (5)$$

If we define both the expansion coefficients

$$\beta_t = \frac{V_t - V_0}{V_0 \cdot t} = \frac{\varrho_0 - \varrho_t}{\varrho_t \cdot t} \quad \text{and} \quad \beta_F = \frac{V_F - V_0}{V_0 \cdot t_F} = \frac{\varrho_0 - \varrho_F}{\varrho_F \cdot t_F} \qquad (6)$$

and substitute them in Eq. (5), the relationship between the Celsius temperature t and the measured volume V_N is seen to be:

$$\frac{1}{1 + \beta_t \cdot t} = \frac{V_0}{V_E} - \frac{V_N - V_E}{V_E} \, \frac{1}{1 + \beta_F \cdot t_F}. \qquad (7)$$

Since the expansion coefficients β_t of linseed-oil over the relevant temperature range are not available in the literature, they have been measured for this paper and the results are shown in Fig. 2. Linseed-oil 1 is designed for

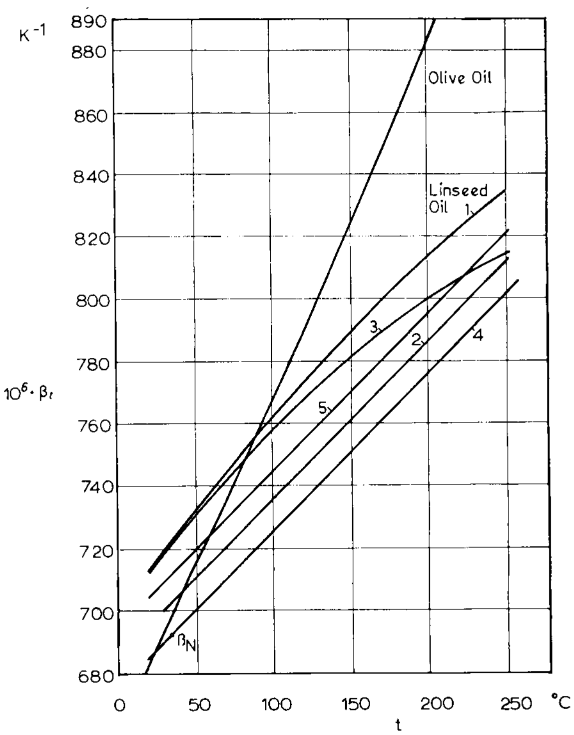

Fig. 2. Expansion coefficients of linseed-oil and olive oil

painting purpose and is clear and of a light yellow hue. Linseed-oil 2 is initially of the same quality as linseed-oil 1, but is tempered at increasing temperatures for a period of 70 hours. In this process the liquid remains clear, but becomes dark brown. Linseed-oil 3 is specified for medical purposes and is, like linseed-oil 1, light yellow and clear. If we plot the value $\beta_N = 692 \cdot 10^{-6}\,\mathrm{K}^{-1}$, measured by Newton at the temperature $t = 37\,°\mathrm{C}$, in Fig. 2, and draw a curve through this point parallel to Curve 2, we arrive at Curve 4. We may assume that Newton's linseed-oil behaved approximately according to this curve. β_N is an expansion coefficient measured relative to glass. If we add to the values of Curve 4 the expansion coefficient of glass ($\approx 20 \cdot 10^{-6}\,\mathrm{K}^{-1}$), we then get the absolute expansion coefficient of linseed-oil (Curve 5). This lies between our experimental results. In Fig. 2 are also to be found the values for olive oil, available from the literature.

In order to find the thread temperature t_F we assume that at the point of immersion the thermometer has the same temperature t as the bath, and that heat in the thermometer is transmitted by conduction and lost to the surroundings. Because of the low thermal conductivity of linseed-oil ($\lambda = 0.17$ W/K m), heat is transmitted almost solely via the glass capillary ($\lambda = 1$ W/K m). Given realistic assumptions, we may observe that the exposed thread has, within only a few centimeter, the same temperature as its surroundings. Our calculations are carried out with the thread temperature $t_F = 25\,°\mathrm{C}$.

Given these assumptions, in Eq. (7) we finally obtain the relationship between the Celsius temperature t and the volume of linseed-oil V_N, and using Eq. (4) also the Newton temperature t_N, as shown in Fig. 3. For the immersed volume the value $V_E = 9550$ was taken. From this, the differences between the temperatures t and t_N for the fixed-points A to M (tin-point) are satisfactorily reproduced.

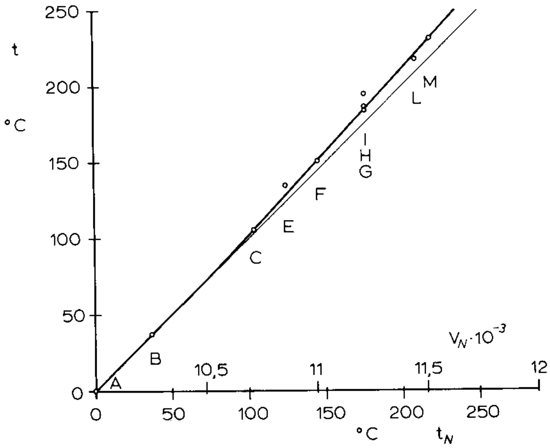

Fig. 3. Newton temperature and Celsius temperature, measured using the linseed-oil thermometer

4 The calorimeter

To measure the higher range of temperatures Newton used a "sufficiently thick piece of iron" (ferrum satis crassum), that was heated until red-hot and then exposed to the wind. Onto this piece of iron, small samples of the metals and alloys were placed, which solidified on cooling. He measured the solidification times.

In order to determine the temperature from this, Newton assumed the heat flux from the iron to the adjacent flow of air to be proportional to the momentary difference of temperature between the iron and the air.

This is "Newton's Law of Cooling". He further sets the transferred heat equal to the decrease of enthalpy of the iron. This corresponds to the First Law. This assumes that the iron is cooled only by convection and not by any further mechanism, for example, radiation. In this way the iron was used as a calorimeter, whose decrease of enthalpy over time is determined by Newton's law of cooling. This process is expressed in the following equation:

$$\Phi\, d\tau = -M\, dh = -M c\, dt = \alpha_1 A\, (t - t_\infty)\, d\tau \tag{8}$$

where Φ is the heat flux, τ the time, and for the iron h is the specific enthalpy, M the mass, c the specific heat capacity, and A the surface exposed to the wind. t_∞ is the temperature of the surrounding air, α the heat transfer coefficient, which is assumed here to be constant and labelled α_1 in Eq. (8), in order to distinguish it from later-used terms. The right-hand side of Eq. (8) may be integrated to give the following:

$$\ln \frac{t - t_\infty}{t_1 - t_\infty} = \frac{\alpha_1 A}{M c}(\tau_1 - \tau), \tag{9}$$

where τ_1 is the total time of the experiment, that is, until the iron has cooled to the temperature t_1. The expression $\alpha_1 A/Mc$ with the dimension of a reciprocal time, is also known as the "time constant".

In reality the heat transfer coefficient α cannot be assumed to be constant over a wide range of temperature differences, nor can the influence of radiation be neglected. Utilising the usual linearisations we therefore expand Eq. (9) to the form:

$$\ln \frac{t - t_\infty}{t_1 - t_\infty} = \frac{\alpha_K + \alpha_S}{\alpha_1} \frac{\alpha_1 A}{M c}(\tau_1 - \tau), \tag{10}$$

where α_K is the heat transfer coefficient of convection dependent on temperature and α_S the heat transfer coefficient of radiation. The latter is defined by the equation:

$$\alpha_S = \varepsilon\, \sigma\, (T^4 - T_\infty^4)/(t - t_\infty), \tag{11}$$

where σ is the Stefan-Boltzmann-Constant and ε the emission coefficient. α_S is strongly dependent on temperature. When $\tau \to \tau_1$ then also $(\alpha_K + \alpha_S) \to \alpha_1$. For the calculation of α_K an equation for surfaces subject to turbulent airflow was applied; these values were further increased

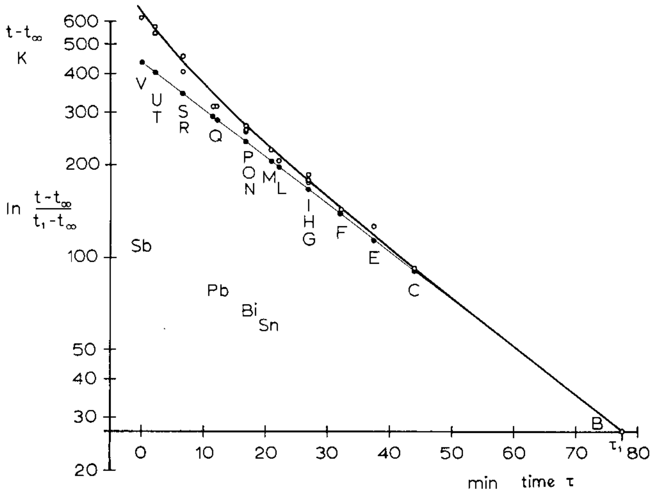

Fig. 4. Cooling of the calorimeter in semi-logarithmic presentation

by 10% to take account of suspension. The emission coefficient ε was set to be 0.55. The surrounding temperature t_∞ was taken as 10 °C, since the experiments were conducted in the open air.

Figure 4 shows the results of the application of Eq. (10). Here $\ln[(t-t_\infty)/(t_1-t_\infty)]$ is plotted against the time τ; on the ordinate $(t-t_\infty)$ is shown. Following Newton's theory the cooling curve for iron is a straight line, as shown in Fig. 4, the equivalent of Eq. (9). The real cooling curve for iron, expressed in Eq. (10), is shown to be a curved line which we have obtained by graphical construction. It can be seen that Newton's measured values for the higher range of temperatures may also be satisfactorily reproduced.

5 Concluding remarks

Amongst the fixed-points which Newton used for the definition of this temperature scale, are more than 20 which are easily reproduced and whose Celsius temperatures can be determined. These Celsius temperatures are higher than the equivalent Newton temperatures, the differences also increasing with temperature. These differences can be understood if we evaluate Newton's experimental results according to our current knowledge. In this respect, for the linseed-oil thermometer both the exposed thread and the temperature dependence of the expansion coefficient are significant, whilst for the calorimeter the principle factor is heat radiation. When considering Newton's experimental results, it is important to recognize that with his arrangements the corrections in the overlapping range of both thermometers have the same direction and approximately the same values.

Newton's motive for undertaking this unique work in thermometry is hard to establish. However, it is not unlikely that Newton's alchemical experiments inspired him to regard the melting- and freezing-points as fixed-points, and to base upon these a temperature scale.

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