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# HEAT TRANSFER IN COMPRESSIBLE TURBULENT BOUNDARY LAYER

K. Bauer[1], W. Winkler[2], U. Grigull[1]

1. Lehrstuhl A für Thermodynamik, Technische Universität D-8000 München, P. O. Box 20 24 20, West Germany

2. Gesellschaft für Reaktorsicherheit (GRS) D-8046 Garching, West Germany

## ABSTRACT

A rectangular unsymmetrical convergent - divergent nozzle is built up with which integral measurements of heat transfer through a two-dimensional compressible boundary layer can be carried out very accurately.

With this nozzle the influence of favorable pressure gradient dp/dx on heat transfer through the boundary layer attached to an isothermal wall has been investigated.

The measured data are compared to measurements and predictions of other authors. These predictions are not satisfactory.

From the own measurements a simple relation is derived for the heat transfer coefficient $\alpha_r$ depending on the favorable pressure gradient dp/dx valid for the subsonic and the supersonic flow in the nozzle up to Ma = 2.2:

$$\alpha_r = C_A + C_B \cdot \log \left(-\frac{\Delta p}{\Delta x}\right)$$

where $C_A$ and $C_B$ are functions of the stagnation pressure $p_o$ for subsonic flow and constant for supersonic flow.

## NOMENCLATURE

Besides those symbols recommended for the papers of the Sixth International Heat Transfer Conference the following one are used in this paper:

| | | |
|---|---|---|
| $A_1$ | = | see eq. (13) |
| $A_2$ | = | |
| B | = | width of the rectangular nozzle, m |
| $C_A$ | = | see eq. (12) |
| $C_B$ | = | see eq. (12) |
| f | = | shape factor, m |
| r | = | recovery factor |
| $R_a$ | = | gas constant of air, 287 J/kg K |
| $R_c$ | = | nozzle throat radius of curvature |
| $\Delta x$ | = | length of segment in flow direction, mm |
| $\sigma$ | = | dimensionless property correction factor (defined in (5)) |

## Subscripts

| | | |
|---|---|---|
| D | = | referred to the equivalent diameter |
| r | = | based on recovery temperature |
| w | = | at the wall |
| $\infty$ | = | freestream condition |
| O | = | total condition |
| * | = | critical condition in the nozzle flow |
| sub | = | subsonic |
| sup | = | supersonic |

## INTRODUCTION

Compressible turbulent boundary layers with heat transfer have to be considered in gas flows with high velocities and high temperatures along cooled walls, for example in hot gas flows through turbine guide vanes, thrust nozzles, and rocket nozzles. In all these cases the heat transfer from the hot gas to the wall should be as low as even possible to decrease the amount of cooling. Therefore, it is to be clarified by which parameters the compressible turbulent boundary layer and the heat transfer through it is influenced.

A few authors /(1)... (10)/ have already worked on that problem. But there is no systematical investigation refering to the whole phenomena and suitable and practical calculation methods are still missing.

To fill this gap we experimentally and theoretically investigated the rate of heat transfer through a two-dimensional compressible turbulent boundary layer, attached to a wall.

The aims of our project have been:

- to make measurements without the evident defects of earlier measurements of other authors, for instance to avoid strong and different wall temperature gradients up to 20 K/cm /see (1)÷(10)/.
- to make heat transfer data available to compressible turbulent boundary layers even in supersonic flows (up to Ma ≈ 2.2).
- to investigate the quantative influence of several parameters as of the favorable and adverse pressure gradient in flow direction dp/dx, the wall temperature gradient in flow direction dT/dx, and the freestream turbulence on the heat transfer through the compressible turbulent boundary layer.
- to give integral relations which cover the measurements.

This paper deals primarily with the influence of favorable pressure gradient dp/dx on the rate of heat transfer through the two-dimensional compressible turbulent boundary layer for an isothermal wall and it is based on our own measurements.

## EXPERIMENTAL EQUIPMENT

An unsymmetrical rectangular convergent - divergent (C - D) nozzle is used in the investigation (Figure 1). It is supplied with hot air from a stationary driven radial compressor. To make very accurate measurements of the heat flux from the hot air to the water cooled nozzle walls the C - D nozzle was built up with 35 seperate segments of copper on each of the two sides. The width of the segments in flow direction is lower or equal to 27 mm. The nominal throat area is 55 mm x 99mm.

The segments are seperated from each other by air gaps and are only connected on the surface by thin VA-tubes. Therefore, lateral heat conduction between the segments is negligible.

The required isothermal wall condition is realized by the individual water flow cooling each segment.

The steady state heat fluxes into the nozzle wall are measured by heat flux meters using NiCr - Co thermocouples (Figure 2). The static pressure is measured by static pressure holes in the VA-tubes on the nozzle surface. These holes are situated in the middle between the two slabs on both sides of the rectangular channel.

In front of the C - D nozzle the total temperature and pressure, the static pressure and the turbulence (using a high temperature hot wire probe) are measured.

Since the distance between the slabs on both sides is 99 mm and the data are taken in the middle between them the two-dimensional flow assumption seems to be a suitable approximation.

To get the turbulent boundary layer in the nozzle as soon as possible, a turbulence wire is put on the test walls just behind the leading edge.

For more details about the experimental apparatus and the test setup see reference (11).

## MEASURED AND CALCULATED DATA

Although measurements have been taken on both test walls only those of the straight wall will be presented in this paper since the results of the curved wall give no further information.

The covered range of compressible flow is described by the freesteam Mach number $Ma_\infty$ (Figure 3) calculated from

$$Ma_\infty = \frac{u}{a} \tag{1}$$

where:

$$u = \sqrt{2\,\frac{\gamma}{\gamma-1} \cdot R_a\,(T_o - T_\infty)} \tag{2}$$

$$a = \sqrt{\gamma \cdot R_a \cdot T_\infty} \tag{3}$$

$$\gamma = 1.4; \quad R_a = 287\ \mathrm{J/kg \cdot K}$$

Assuming that the heat transfered to the nozzle wall is generated by dissipation in the boundary layer and not withdrawn from the freestream, the measured total temperature $T_o$ is constant through the nozzle and the local static freestream temperature T is calculated with the knowledge of the freestream static-to-stagnation pressure ratio along the nozzle $p_\infty/p_o = f(x)$ from the isentropic equation:

$$T_\infty = T_o \left(\frac{p_\infty}{p_o}\right)^{\frac{\gamma-1}{\gamma}} \tag{4}$$

By the assumption dp/dy = 0 across the boundary layer the freestream static pressure $p_\infty$ is equal to the static pressure on the wall $p_w$. The measured static-to-stagnation pressure ratio $p_\infty/p_o = f(x)$ is given in Figure 4 for several loading conditions covering a range of stagnation pressure from 1.02 bar to 2.77 bar. From $p_\infty$ /po the favorable pressure gradient

$dp/dx = \Delta p/\Delta x$ is calculated (Figure 5). The curves are cut off at the shock behind which an adverse pressure gradient $\Delta p/\Delta x$ exists; that is not a concern of this paper.

It should be noted that all data have been plotted without any smoothing.

With the measured heat fluxes the following parameters are calculated which are generally used to describe the heat transfer:

- heat transfer coefficient $\alpha_r$ related to the recovery temperature (Figure 6 and 8)

$$\alpha_r = \frac{\dot{Q} / B}{(T_r - T_w) \cdot \Delta x} \quad (5)$$

where:

$$\dot{Q} = \lambda_{Cu} \cdot f \cdot (T_w - T_K) \quad (6)$$

$$T_r = T_\infty + r\,(T_o - T_\infty) \quad (7)$$

$r = Pr^{1/3}$ (see reference (12))

$Pr = 0.7$ (in the interesting temperate range)

$T_\infty$ from equation (4)

- Stanton Number (Figure 9) from

$$St = \frac{\alpha_r}{\rho_\infty \cdot u_\infty \cdot c_p} \quad (8)$$

where $\rho_\infty$ comes from the thermal equation of state for a perfect gas

$$\rho_\infty = p_\infty / (R_a \cdot T_\infty) \quad (9)$$

## COMPARISON WITH OTHER MEASUREMENTS AND PREDICTIONS

Our data are in good agreement with those of Back, Massier and Gier (4) from an axial symmetric nozzle with the same convergent (30°) and divergent (15°) half-angle as in our nozzle. Corresponding to ref. (4) we found that the pipe flow equation

$$St \cdot Pr^{0.6} = 0.023 \cdot Re_D^{-0.2} \quad (10)$$

as well as the equation of Bartz (5)

$$\alpha = \left[ \frac{0.026}{(D_h^*)^{0.2}} \cdot \left(\frac{\eta^{0.2} \cdot c_p}{Pr^{0.6}}\right)_o \cdot \left(\frac{\dot{M}}{A^*}\right)^{0.8} \cdot \left(\frac{D_h^*}{R_c}\right)^{0.1} \right] \cdot \left(\frac{A^*}{A}\right)^{0.9} \cdot \sigma \quad (11)$$

do not fit the measurements satisfactorily (Figure 7). While eq. (10) under-predicts the subsonic heat transfer and over-predicts the supersonic one, each in the order of 20 to 30 per cent, eq. (11) describes our measurements quite well in the subsonic part of the nozzle (the deviation is only about 5 per cent), but not so in the throat and in the supersonic region (10 to 15 per cent overpredicted). The main defect is that the maximum of the heat transfer coefficient is predicted too far downstream.

Equations of others authors, e.g. Moretti and Kays (6), have also been looked at. These equations, however, fail in describing our measurements in most part of the investigated pressure gradient range too.

## PRESENTATION OF THE OWN RESULTS

To show the influence of favorable pressure gradient dp/dx on the heat transfer through the boundary layer explicitly, the heat transfer parameters $\alpha_r$ (eq. 5) and St (eq. 8) have been plotted versus dp/dx in Figure 8 and 9. Looking at Figure 8 one can realize a linear dependence of $\alpha_r$ on $\log(\Delta p/\Delta x)$ for subsonic and supersonic flow. $\alpha_r$ seems to be independent of stagnation pressure $p_o$ for supersonic flow but not so for subsonic flow, where the heat transfer rate increases with increasing $p_o$. The observed trend in the results can be approximated within experimental uncertainty by the following relation:

$$\alpha_r = C_A + C_B \cdot \log\left(-\frac{\Delta p}{\Delta x}\right) \quad (12)$$

where for subsonic flow ($Ma \leq 1$):

$$C_A = -C_B \cdot \log\left(-\frac{\Delta p}{\Delta x}\right)_{sub} \quad \text{(s.Fig.8)} \quad (13a)$$

$$C_B = C_B\,(p_o) \quad \text{(s.Fig.10)} \quad (14a)$$

and for supersonic flow ($Ma \geq 1$):

$$C_A = -\log\left(-\frac{\Delta p}{\Delta x}\right)_{sup} \quad (13b)$$

(s.Fig.8)

$$C_B \neq C_B\,(p_o) \quad (14b)$$

Plotting for the subsonic flow $C_B$ versus $p_o$ (excluding the data just behind the leading edge) we find the simple linear relation (Figure 10)

$$C_B = A_1 \cdot p_o + A_2 \quad (13)$$

Near the leading edge (subsonic) and near the shock wave (supersonic) eq. (12) has quite another slope as for the main part of the nozzle and $C_B$ is no longer dependent on $p_o$ for the subsonic condition (Figure 10, lower curve). The slope, however, is the same for the subsonic curves and the supersonic only one and is equal to the shifting parameter $A_2$ of eq. (13). The reason for the changed behaviour seems to be a non-turbulent boundary layer (laminar or in transition behind the leading edge, relaminarisation by shock interaction with the boundary layer). We call that slope $C_{B,lam}$.

$$C_{B,lam} = A_2$$

Since only the turbulent boundary layer is a concern of our investigation we will not go more into detail for the heat transfer in the non-turbulent part of the boundary layer.

In addition to the relation $\alpha_r = \alpha_r(\Delta p/\Delta x)$ the dependence of the Stanton Number on $\Delta p/\Delta x$ is given in Figure 9. The vertex of the curve dividing the subsonic and the supersonic flow seems to shift to slightly higher St-Numbers and the aperture of curves increases with rising stagnation pressure $p_o$. The functional connection of St-Number and $\Delta p/\Delta x$ is not given in this paper since it is not finally worked out. It should be noted, however, that the St-Number can be given by one relation depending on $\Delta p/\Delta x$ valid for subsonic and supersonic flow condition up to $Ma_\infty = 2.2$.

REFERENCES


1. Welsch, W.E. and Witte, A.B., Trans. ASME C, J. Heat Transfer 84, p. 19 - 28, 1962

2. Witte, A.B. and Harper, E.Y., AI AA J., Vol. 1, p. 443 - 451, 1963

3. Fortini, A. and Ehlers, R.C., NASA TN D - 1743, 1963

4. Back, L.H., Massier, P.F. and Gier, H.L. Int. J. Heat Mass Transfer, Vol 7, p. 549 - 568, 1964

5. Bartz, D.R., Jet Propulsion, Vol. 27, p. 49 - 51, 1957

6. Moretti, P.M. and Kays, W.M., Int. J. Heat Mass Transfer, Vol. 8, p. 1187 - 1202, 1965

7. Boldman, D.R., Schmidt, J.F. and Fortini, A., NASA TN D-3221, 1966

8. Boldman, D.R., Graham, R.W., NASA TN D-6594, 1972

9. Boldman, D.R., Schmidt, J.F. and Ehlers, R.C., NASA TN D-6595, 1972

10. Brott, D.L., Yanta, W.J., Voisinet, R.L. and Lee, R.E., AIAA J., Vol. 8, p. 1270 - 1274, 1970

11. Winkler, W., Wärmeübergang turbulenter kompressibler Grenzschichtströmungen mit starken negativen Druckgradienten (Wärmeübergang in einer Lavaldüse), Dissertation, TU München 1976

12. Cebeci, T. and Smith, A.M.O., Analysis of Turbulent Boundary Layers, Academic Press 1974


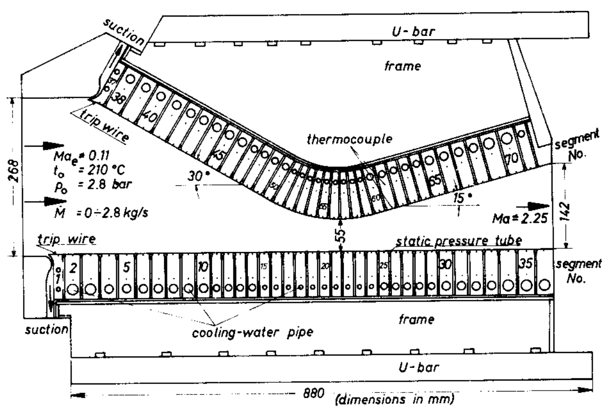


Figure 1 Longitudinal Section of the Rectangular C-D Nozzle

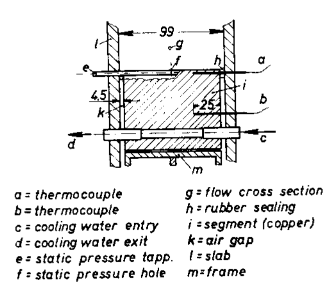


Figure 2 Cross-Section of One Segment from the C-D Nozzle

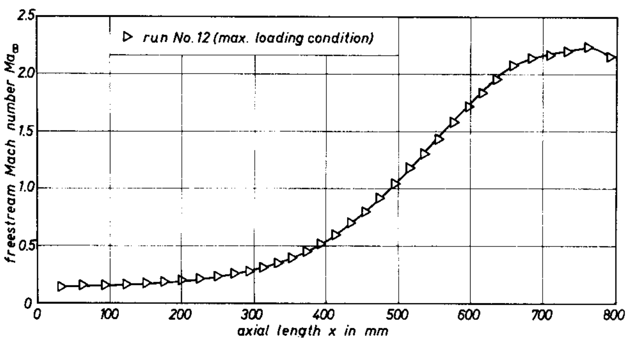


Figure 3 Freestream Mach Number $Ma_\infty$ vs. Axial Length x

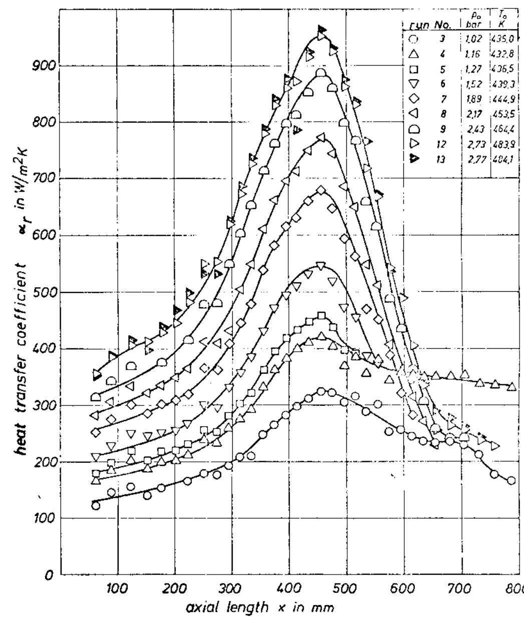


Figure 6 Heat Transfer Coefficient $\alpha_r$ vs. Axial Length x (Isothermal Wall)

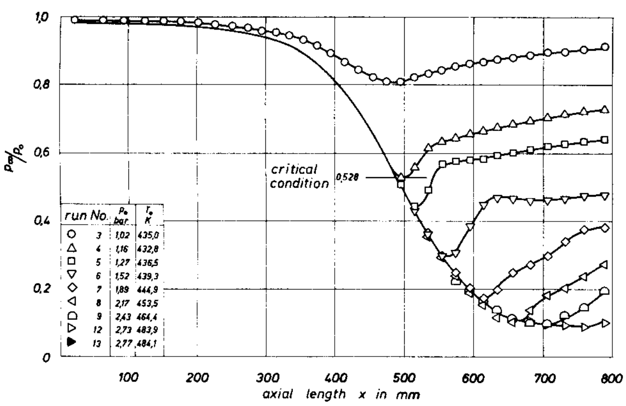


Figure 4 Static-to-Stagnation Pressure Ratio $p_\infty/p_o$ vs. Axial Length x

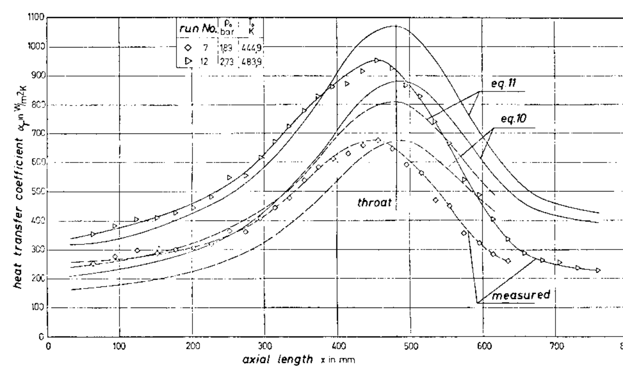


Figure 7 Comparison of Predictions and Own Measurements for Heat Transfer Coefficient $\alpha_r$ vs. Axial Length x

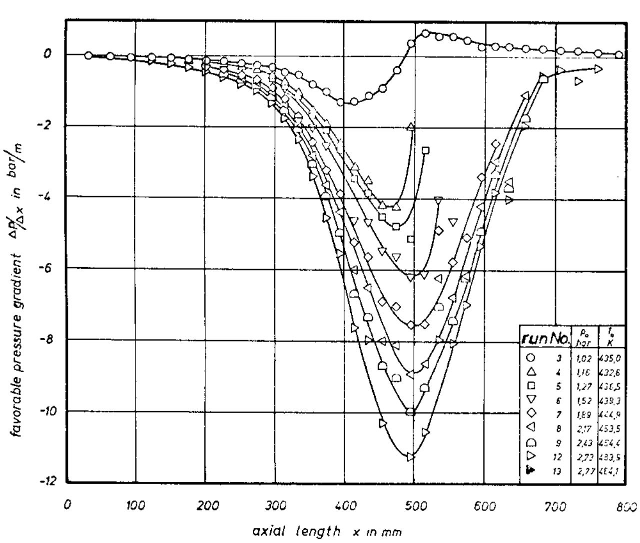


Figure 5 Favorable Pressure Gradient $\Delta p/\Delta x$ vs. Axial Length x

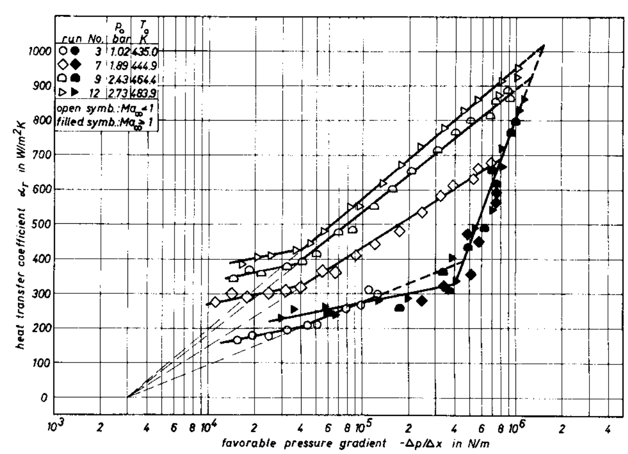


Figure 8 Heat Transfer Coefficient $\alpha_r$ vs. Favorable Pressure Gradient $\Delta p/\Delta x$ (Isothermal Wall)

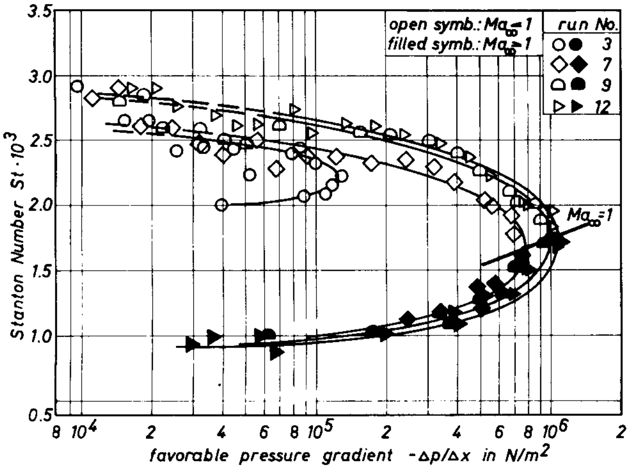


Figure 9 Stanton Number St vs. Favorable Pressure Gradient $\Delta p/\Delta x$ (Isothermal Wall)

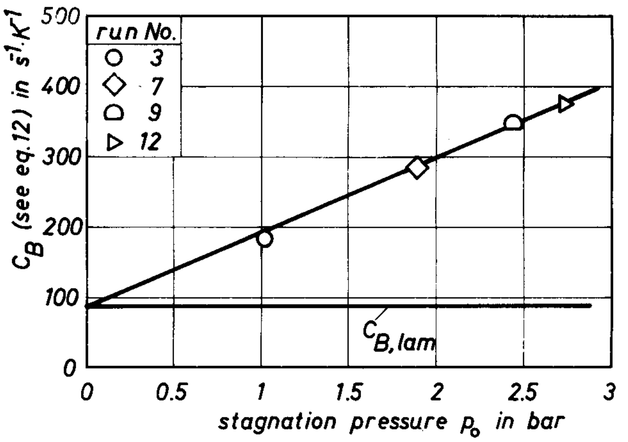


Figure 10 Coefficient $C_B$ of the Relation $\alpha_r = \alpha_r$ $(\Delta p/\Delta x)$ vs. Stagnation Pressure $p_o$. (Isothermal Wall)