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THE REGELATION OF ICE—A PROBLEM OF HEAT CONDUCTION

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Abstract—In the classical experiment for the regelation of ice, a weight loaded wire sling is made to pass through a block of ice, but does not cut it apart because the plane of separation refreezes again.

The usual explanation is only based on the anomaly of water, specifically the decrease of melting temperature with an increase in pressure; as stated in the Clausius–Clapeyron equation. It is shown here that heat conduction plays a dominant part in this experiment.

The classical experiment was repeated with a variety of wires of different thermal conductivity and diameter, and various pressures were applied.

From a simplified model for the heat conduction mechanism an equation for the prediction of the penetration rate could be obtained. The agreement of this theoretical equation with the experimental results is very good compared to other work. For metallic wires, the assumption of a linear temperature-distribution around the wire gives better agreement than sinusoidal.

Perlon threads penetrate faster through the ice than theory predicts, thus indicating unseizable defects in the theory. An interlayer influence of the material–ice combination, dominating the penetration rate, could not be observed.

NOMENCLATURE

- a, radius of wire;
- h, latent heat of fusion;
- k, thermal conductivity;
- L, effective length of wire;
- Δp_0, maximum excess pressure;
- Q_{ci}, heat conducted through ice;
- Q_{cm}, heat conducted through wire material;
- Q, heat required for melting;
- r, radial distance in cylindrical coordinates;
- Δs, length interval of penetration in the ice;
- T, temperature;
- ΔT_0, maximum temperature difference (Clausius–Clapeyron equation);
- Δt, time interval of penetration;
- v', specific volume, liquid state;
- v, specific volume, solid state;
- w, penetration speed;
- δ, water layer thickness;
- ϑ, temperature excess;
- ρ, density;
- ϕ, heat flux;
- φ, angle in cylindrical coordinates;
- μ, dynamic viscosity.

Indices

- e, experiment;
- i, ice;
- l, linear case;
- m, wire material;
- n, arbitrary material;
- s, sinusoidal case;
- t, theory;
- w, water.

1. INTRODUCTION

THE CLASSICAL demonstration experiment is performed with a weight loaded wire, cutting its way through a block of ice but leaving it undivided. The usual explanation is given with a melting of ice below the wire due to the increase in pressure and a refreezing above the wire where the pressure is released. Thus, regelation appears to be a problem of thermodynamic stability, only depending on pressure changes. Many textbooks adhere to this explanation although as early as 1906, Ornstein [1] has derived a formula for the penetration rates of wires through ice, based on a theory of regelation. This formula was tested by Meerburg [2] with metallic wires. Descrepancies between theory and experiment came up to 500 per cent.

In recent years Nye [3] described an elaborate "theory of regelation" which in physical and mathematical concept is similar to Ornstein's. This theory was connected to experiments by Townsend and Vickery [4] on metallic discs and spheres, and other experiments by Nunn and Rowell [5] on metallic wires and Nylon or linen threads.

Comparison between these experiments and this theory revealed discrepancies up to 3300 per cent (Townsend/Vickery) and up to 1800 per cent (Nunn/Rowell), so that both, experimental and theoretical results appear in doubt.

In a "supplementary note" it is argued by Frank [6] that instabilities in the geometry of the waterlayer around the wire may explain the experimental results.

In our work which was started in spring 1967 without knowledge of the above mentioned investigations, the classical experiment was performed with wires of different materials (silver, copper, iron, Perlon) with different diameters at different bearing pressures.

2. EXPERIMENTS

2.1 *Experimental procedure*

A detailed description of the experimental set up as well as the extremely careful preparation of the ice was reported before by Hahne/Grigull [7].

The penetrating wires were held in a rigid wooden frame, loaded with weights according to the excess pressure desired. The criterion for a constant quality ice was its transparency, Fig. 1. The dome-shaped cavity at the bottom indicates the pond left behind in the ice during the freezing process. The outer black lines give the block boundaries.

No air inclusions or other visible contaminations were allowed. In early experiments it had been observed that air bubbles in the ice cause a wide scatter in the results and that these results are always below those for ice without bubbles.

As the experimental result, the speed of penetration of the wires through a block of ice (40 × 12 × 12 cm) was observed. The penetration speed of each wire of a certain material, a certain diameter, and a certain pressure was observed for at least ten times. The silver- and copper-wire experiments were performed many more times.

Immediately before the test, the wires were carefully cleaned with acetone and rinsed with distilled water. Each wire was used only once. Again, in earlier experiments it was observed that deposits formed on the metallic wires if they were used repeatedly. These deposits caused a scatter in results and a decrease in speed.

Before the experiment, the block of ice was stored for 45 min. at about 20°C, to secure a 0 °C temperature throughout its entire volume. A special consideration was made for the heat conducted along the wire from the surroundings into the ice by the fin effect: The effective bearing length of the wire was observed by subtracting the lengths influenced by the fin effect from the ice block width. The melting of the block around its circumference during a test run caused a pressure increase of about 10 per cent. The pressures indicated are mean pressures of a run.

2.2 *Results*

2.2.1 *Bare materials.* Copper and Perlon (a Nylon-type plastic material) were selected for their big difference in thermal conductivity which amounts to three orders of magnitude (Table 1).

In Fig. 2, the results are presented in a plot of experimental penetration rate w_e vs. wire radius a, with the mean excess pressure p, exerted by the wire, as a parameter.

Experimental results shown here, were obtained for copper wires with a radius of 0·25, 0·4 and 0·5 mm at pressures of 5, 7 and 10 bar and for Perlon strings with radii 0·12, 0·17 and 0·24 mm at pressures of 5, 7 and 8·5 bar. Each

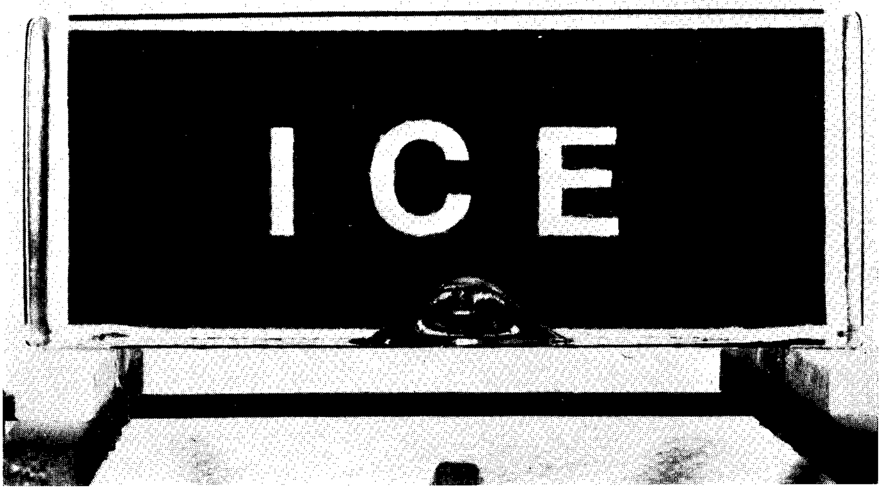

FIG. 1. Transparent block of ice in front of a black background.

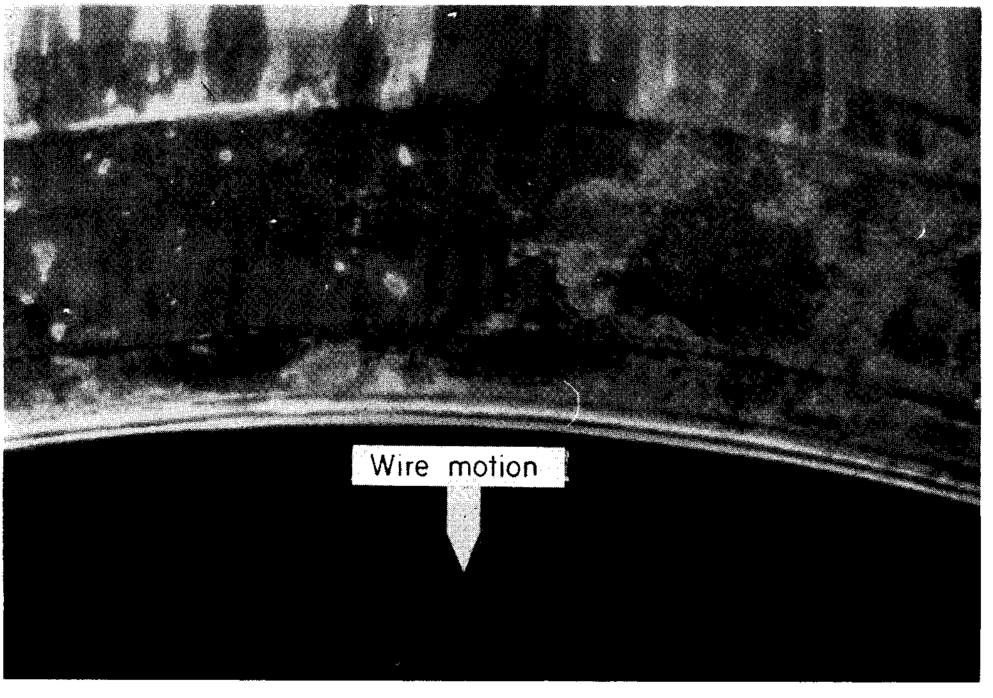

FIG. 4. Water layer around the wire.
Unregelated ice: lower part of the picture
Regelated ice: upper part.

Table 1.

Material	k (W/mK)	radius a (10^{-3} m)	pressure, p (bar)	experim. penetr. rate w_e (mm/h)	Linear case δ_l (10^{-6}m)	w_{tl} (mm/h)	w_{tl}/w_e	Sinusoidal case δ_s (10^{-6}m)	w_{ts} (mm/h)	w_{ts}/w_e
Silver	419	0·50	10	59 ± 3	0·82	82·35	1·40	0·88	100·34	1·70
Copper	350	0·25	5	28 ± 3	0·59	62·15	2·22	0·64	75·71	2·70
		0·25	7·5	46 ± 2·5	0·59	93·22	2·03	0·64	113·56	2·47
		0·25	10	70 ± 3	0·59	124·30	1·78	0·64	151·41	2·16
		0·40	5	22 ± 3	0·73	44·86	2·04	0·78	54·79	2·49
		0·40	7·5	37 ± 2	0·73	67·29	1·82	0·78	82·18	2·22
		0·40	10	57 ± 3	0·73	89·72	1·57	0·78	109·57	1·92
		0·50	5	20 ± 3·5	0·80	38·36	1·92	0·86	46·85	2·34
		0·50	7·5	34 ± 3	0·80	57·54	1·69	0·86	70·27	2·07
		0·50	10	54 ± 3	0·80	76·72	1·42	0·86	93·69	1·74
Copper	350	0·25	10	68 ± 4	0·59	122·63	1·80	0·63	149·42	2·20
silverplated 10 μm	419	0·50	10	51 ± 4	0·80	76·09	1·49	0·86	92·94	1·82
Copper	350	0·25	10	19 ± 2	0·26	10·26	0·54	0·28	12·95	0·68
varnished 15 μm	0·6									
Iron	58	0·25	10	44 ± 2	0·42	44·95	1·02	0·46	56·12	1·28
Iron	58	0·25	10	38 ± 2	0·42	44·73	1·18	0·46	55·82	1·47
silverplated 5 μm	419									
copper interlayer 5 μm	350									
Iron	58	0·25	10	16 ± 2	0·25	9·36	0·59	0·27	11·68	0·73
varnished 15 μm	0·6									
Perlon	0·3	0·12	5	9 ± 0·5	0·12	2·26	0·25	0·13	2·86	0·32
		0·12	7·5	14·0 ± 0·5	0·12	3·39	0·24	0·13	4·29	0·31
		0·12	8·5	17·0 ± 0·5	0·12	3·84	0·23	0·13	4·86	0·29
		0·17	5	6·2 ± 0·3	0·14	1·62	0·26	0·15	2·05	0·33
		0·17	7·5	10·0 ± 0·3	0·14	2·42	0·24	0·15	3·07	0·31
		0·17	8·5	11·0 ± 0·3	0·14	2·75	0·25	0·15	3·48	0·32
		0·24	5	4·5 ± 0·2	0·15	1·14	0·25	0·17	1·44	0·32
		0·24	7·5	6·5 ± 0·3	0·15	1·70	0·26	0·17	2·16	0·33
		0·24	8·5	7·5 ± 0·5	0·15	1·93	0·26	0·17	2·45	0·32
		0·24	10	9·3 ± 0·3	0·15	2·27	0·24	0·17	2·88	0·31

circle indicates a mean value from at least ten observed single velocities and the shaded area gives the statistic scatter with 95 per cent probability.

The difference in the penetration speed of the two materials with different thermal conductivity k is striking. The tendencies of the curves, however, are the same: an increase in radius causes a decrease in speed.

The scatter in the results for Perlon is considerably smaller than that for copper.

2.2.2 *Coated materials.* The assumption that there always exists a very thin, water-like film on the surface of ice as well as in any gap between ice and some other material—which was later proved by Fletcher [8], Bullemer/Riehl [9], Nakamura [10]—had led to the hypothesis by Weyl [11] that the different penetration speeds for different materials should be explained on the basis of different film thicknesses rather than different thermal conductivities.

In order to examine this reasoning, copper- and iron wires were silver-plated and their speed compared to that of pure silver. In another comparison, insulating varnish as a poor thermal conductor (k = 0·6 W/mK) was used for

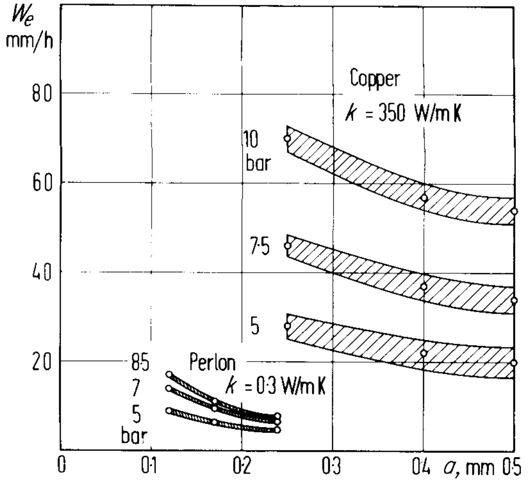

FIG. 2. Penetration speed of copper—and Perlon—wires for different radii a at various pressures.

material–ice combination would dominate the regulation phenomenon, the velocity of bare silver- and silver-plated copper-wires should be the same for corresponding diameters and pressures. The difference in velocity of bare copper- and iron wires is maintained, however, for silver-plated materials. Thus thermal conductivity is dominant.

In the case of a varnish-coating for copper and iron, also a difference in penetration rate is observed, but this is much smaller now. For comparison a Perlon result ($a = 0{\cdot}24$ mm. $p = 10$ bar) is added. Perlon has about the same conductivity ($k = 0{\cdot}3$ W/mK) as insulating varnish, so again, the decrease in speed is due to the decrease in thermal conductivity of the coated wires.

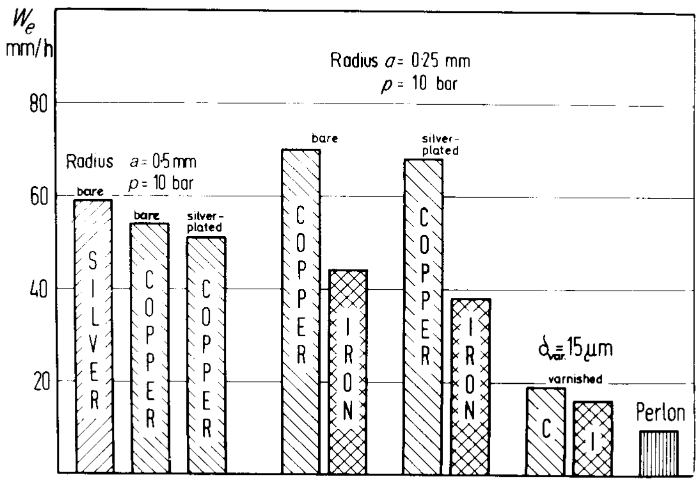

FIG. 3. Penetration speed of different materials, bare, silver-plated and varnished. Constant pressure at two different radii.

coating copper and iron. The silver plating was 10 μm thick on copper, and 5 μm on iron with an intermediate copper layer of 5 μm. The varnish coating was 15 μm thick. For two constant pressures and two radii the results are shown in Fig. 3. The height of each column represents the mean penetration rate. If the

2.3 *Observations in the experiments*

In all experiments a distinct difference in the structure of the ice in front and behind the moving wire was clearly visible: while the ice in front was completely transparent the refrozen plane behind, maintained a milky, opaque appearance with cloudy irregularities. This difference is shown in Fig. 4. This Fig. 4 is a black and white print of a color photograph in which liquid and solid state can be discerned by color. During the experiments it could be observed that bubbles formed along the wire while it was penetrating through the ice. These bubbles were left behind in the ice while the wire moved away downwards. They vanished about 15 min after their first appearance. Whenever these bubbles appeared, the penetration rate decreased [7].

By dissolving a minimum amount of Fluorescein-Sodium ($C_{20}H_{10}Na_2O_5$) in water, a solution is obtained which can be used as a tracer, because it exhibits luminescence under UV irradiation. With a syringe the cold (0°C) solution was injected along the wire. When the bubbles formed, it was observed that within a fraction of a second, they became filled with the luminescent solution. Even though in daylight it appeared as if the bubbles had completely vanished after a while, a remaining luminescence in UV light revealed that the liquid had not entirely refrozen—there is no luminescence in Fluorescein-Sodium solid state. The irregularities in the regelated plane, shown as white spots here, are thus liquid filled islands in the ice.

As white lines, the tracer method also discloses a water layer of rather uniform thickness beneath the wire and a thicker and very irregular liquid layer above it.

Examination of penetration rate with and without tracer, gave no difference beyond the scatter mentioned before. No systematic drift to lower rates was observed, as might be expected because of depression of melting temperature.

3. THEORETICAL CONSIDERATIONS

3.1 *Model assumption*

The distinct difference in the penetration speed of copper and Perlon as well as in the coated-wire experiments show the dominate effect of thermal conductivity on the penetration mechanism. For a theoretical consideration, a model, as sketched in Fig. 5 will be assumed. While the wire is moving through the ice, the layer immediately below the wire melts. The water containing the heat of fusion, is pressed around the wire to the upper side where it freezes again. In freezing, this heat of fusion is set free and will be conducted to the melting zone where it is needed.

For wire materials with a thermal conductivity greater than that of ice, copper for example, most of the heat will be conducted through the wire. For materials such as Perlon, however, the heat will predominantly be conducted

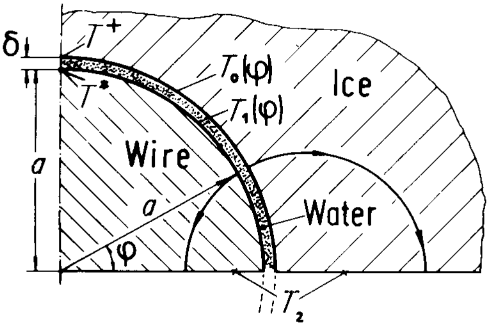

FIG. 5. Model for theoretical considerations.

around the wire through the ice. The driving temperature difference is due to a difference in pressure above and below the wire and can be calculated from Clausius–Clapeyron's equation, which gives for water

$$\Delta T_0 = T(v' - v)\,\Delta p_0/h = -0{\cdot}0074\,\Delta p_0 \quad \text{Kelvin/bar} \qquad (1)$$

For the temperature distribution around the wire, assumptions are made. It is further assumed that the wire is surrounded by a water layer which has to be passed by the heat conducted. This later assumption is of decisive importance in the case of well conducting materials, since the thermal resistivity of such a layer proves to be about the same as that of a metallic wire. For steady state conditions the heat balance will be:

$$Q = Q_{cm} + Q_{ci} \qquad (2)$$

with

$$Q = \pi a L \Delta s \rho_i h \qquad (2a)$$

and

$$Q_{cm} = \phi_m \Delta t\, 0;\ Q_{ci} = \phi_i \Delta t. \tag{2b}$$

3.2 *Heat conduction equations*

For the heat flux through and around the wire the knowledge of the temperature fields is required. It will be obtained by solving the Laplace equation, simplified for negligible end effects

$$r^2 \frac{\partial^2 \vartheta}{\partial r^2} + r \frac{\partial \vartheta}{\partial r} + \frac{\partial^2 \vartheta}{\partial \varphi^2} = 0 \tag{3}$$

with ϑ being an excess temperature to a proper isotherm. Such an isotherm is here the horizontal line of symmetry with the temperature

$$T_2 = [T_1(\varphi) + T_3(\varphi)]/2.$$

$T_3(\varphi)$ being the temperature. as a function of the angle φ, of the leading side of the wire.

The surface temperature of the wire will undergo a steady change from the lowest value at the leading point ($\varphi = -\pi/2$) to the highest value at the trailing point ($\varphi = +\pi/2$). For the boundary conditions it is assumed, therefore, that the temperature along the circumference of the wire follows either a linear distribution or a sinusoidal. The same boundary conditions are assumed for the surface of the ice adjacent to the melting water. Full symmetry is assumed.

Boundary conditions:

Heat conduction through the wire

Linear case	Sinusoidal case
$\vartheta(a, \varphi) = 2\vartheta^* \varphi/\pi$ for $0 \leqslant \varphi \leqslant \pi/2$	$\vartheta(a, \varphi) = \vartheta^* \sin \varphi$ for $0 \leqslant \varphi \leqslant \pi$
$\vartheta(a, \varphi) = 2\vartheta^*(\pi - \varphi)/\pi$ for $\pi/2 \leqslant \varphi \leqslant \pi$	

$$\vartheta(r, 0) = O;\ \vartheta(r, \pi) = O \quad \text{for } 0 \leqslant r \leqslant a \tag{4}$$

Heat conduction through the ice

Linear case	Sinusoidal case
$\vartheta(a + \delta, \varphi) = 2\vartheta^+ \varphi/\pi$ for $0 \leqslant \varphi \leqslant \pi/2$	$\vartheta(a + \delta, \varphi) = \sin \varphi$ for $0 \leqslant \varphi \leqslant \pi$
$\vartheta(a + \delta, \varphi) = 2\vartheta^+(\pi - \varphi)/\pi$ for $\pi/2 \leqslant \varphi \leqslant \pi$	

(5)

$$\vartheta(r, 0) = 0;\ \vartheta(r, \pi) = 0 \quad \text{for } a + \delta \leqslant r \leqslant \infty.$$

The maximum excess temperatures ϑ^* and ϑ^+ are introduced for

$$\vartheta^* = T^* - T_2 \text{ and } \vartheta^+ = T^+ - T_2 = \Delta T_0/2.$$

By separation of variables, straight forward solutions of the Laplace equation are obtained yielding the temperature distributions:

Within the wire, linear case

$$\vartheta_{wl} = \frac{8\vartheta^*}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{r}{a}\right)^n \sin n\frac{\pi}{2} \cdot \sin n\varphi \tag{6a}$$

sinusoidal case

$$\vartheta_{ws} = \vartheta^* \frac{r}{a} \sin \varphi. \tag{6b}$$

Within the ice, linear case

$$\vartheta_{il} = \frac{8\vartheta^+}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{a + \delta}{r}\right)^n \sin n\frac{\pi}{2} \cdot \sin n\varphi \tag{7a}$$

sinusoidal case

$$\vartheta_{is} = \vartheta^+ \frac{a + \delta}{r} \sin \varphi \tag{7b}$$

The heat fluxes ϕ are obtained by integration

$$\phi_{\substack{r=a \\ r=a+\delta}} = L \int_{\varphi=0}^{\pi} k\, r \operatorname{grad} \vartheta \, d\varphi. \tag{8}$$

With the thermal conductivity k assumed to be uniform throughout the material considered, the results are:

through the wire / through the ice

linear case

$$\phi_m = Lk_m \vartheta^* \pi/2 \quad (9a) \qquad \phi_i = Lk_i \vartheta^+ \pi/2 \quad (10a)$$

sinusoidal case

$$\phi_m = 2Lk_m \vartheta^* \quad (9b) \qquad \phi_i = 2Lk_i \vartheta^+ \quad (10b)$$

The water layer surrounding the wire, and the wire-coatings are assumed thin enough to be treated as plane layers. Taking these layers into account, equations (9a, 9b) are extended into

$$\phi_m = La\vartheta^+\pi/2(a/k_m + \delta/k_w + \Sigma\delta n/k_n) \quad (9c)$$

$$\phi_m = 2La\vartheta^+/(a/k_m + \delta/k_w + \Sigma\delta n/k_n). \quad (9d)$$

3.3 *Results*

From the heat balance (2) with (2a), (2b) and the Clausius–Clapeyron equation (1), the penetration rate can be determined as:

$$w_{tl} = \frac{T(v - v')}{4h^2\rho_i}\Delta p_0\left[\frac{1}{a/k_m + \delta/k_w + \Sigma\delta n/k_n} + \frac{1}{a/k_i}\right] \quad (11)$$

in the linear case,
and as

$$w_{ts} = \frac{T(v - v')}{\pi h^2\rho_i}\Delta p_0\left[\frac{1}{a/k_m + \delta/k_w + \Sigma\delta n/k_n} + \frac{1}{a/k_i}\right] \quad (12)$$

in the sinusoidal case.

3.4 *Water layer thickness*

For the prediction of the penetration rate from equations (11) and (12), knowledge on the water layer thickness is lacking. Additional information on this thickness can be gained from the Navier–Stokes equation. In view of the observations described in chapter 2.3, however, this can only be approximative, providing the order of magnitude for δ.

Assuming one-dimensional "creeping flow" (body forces $\ll$ viscous forces) for a constant volume flow rate based on wire length: $\dot{m} = wa\rho_i/2\rho_w$, within a small slit of width δ, with one boundary fixed and the other moving with penetration velocity $w = w_{tl}$ or w_{ts}, the total pressure drop around the wire is obtained as

$$\Delta p_0 = \frac{6\mu aw\pi}{\delta^2}\left[\frac{a\pi}{\delta}\cdot\frac{\rho_i}{\rho w} - 1\right] \quad (13)$$

Here μ is the dynamic viscosity.

Combining equation (13) with (11) or (12) gives for the linear case

$$\frac{6\mu a\pi}{\delta_l^2}\left[\frac{a\pi}{\delta_l}\cdot\frac{\rho_i}{\rho_w} - 1\right] = \frac{4\cdot\rho_i\cdot h^2}{T(v - v')}\cdot\left[\frac{1}{\frac{1}{a/k_m + \delta_l/k_w + \Sigma\delta_n/k_n} + \frac{1}{a/k_i}}\right] \quad (14)$$

and for the sinusoidal case

$$\frac{6\mu a}{\delta_s^2}\left[\frac{a\pi}{\delta_s}\cdot\frac{\rho_i}{\rho_w} - 1\right] = \frac{\rho_i\cdot h^2}{T(v - v')}\cdot\left[\frac{1}{\frac{1}{a/k_m + \delta_l/k_w + \Sigma\delta_n/k_n} + \frac{1}{a/k_i}}\right]. \quad (15)$$

Practically, these third power equations in δ can be solved by trial and error.

4. COMPARISONS

4.1 *Comparison of experimental and theoretical results*

In Table 1 experimental and theoretical results are compiled. The thermal conductivities of the copper and iron-wires used, were measured, others were taken from handbooks. The theoretical water layer thickness determined from (14) or (15) for either case of temperature distribution is listed and a slight dependence of water layer thickness on the material–ice combination can be seen [11], it is not decisive, however.

Comparison of theory and experiment is made by introducing the ratio of theoretical to experimental penetration rate w_t/w_e. Discrepancies in these data go up to 270 per cent for metallic wires and up to 435 per cent for Perlon threads.

For metallic wires a linear temperature distribution along the wire's circumference proves more appropriate; for Perlon threads the sinusoidal case gives better agreement.

With regard to the observations made in 2.3 the theoretical water-layer thickness appears too thin; the irregularities in thickness are not considered. The liquid islands in the regelated region constitute a defect in the heat balance since the respective heat of solidification is missing. All these effects contribute towards a decrease in experimental penetration rate and could explain the deviations.

For Perlon threads, however, the water layer around the wire is not crucial since most of the heat is conducted through the ice. Liquid left-overs apparently have no effect since here experimental velocity always exceeds by far the theoretical. Observation of our polycristalline blocks of ice at melting temperature revealed a fine network within the entire block, probably melted layers along grain-boundaries. Thus it appears likely that the latent heat of melting is not only delivered from the regelating zone, as theory states, but also from other parts of the ice close enough to the melting region. The difference in scatter of experimental data for copper and Perlon (Fig. 2 and Table 1) can indicate the presence of all mentioned effects for copper, while for Perlon the latter only is relevant.

4.2 *Comparison to other investigations*

Ornstein's [1] and Nye's [3] theories are alike in the concept of cosinusoidal pressure— and temperature-distribution around the wire. Meerburg [2] reports on occasional grooves on both sides of the ice and bubbles in the block; both occurrences proved to be interconnected in our observations [7]. For the lack of sufficient data no direct comparison to Meerburg's experiments is possible. In Table 2 theoretical results according to [3] and experimental results by Nunn and Rowell [5] are listed from their "Table of results" [5] and compared to results as obtained from our equations. Physical properties were taken as given in [5].*

Our results give a thicker water layer by about 10 per cent and always lower values of the theoretical penetration rate. The ratio w_{tN}/w_{ts} indicates that Nye's velocities are about twice as high as ours. The sinusoidal case was chosen here for agreement in boundary conditions. Even more advantageous for metallic wires, however, is the assumption of a linear temperature distribution, as comparison to the experimental results of Nunn/Rowell in $w_{tl}/w_{\varrho N}$ leads to about a 2·5 times improvement over $w_{tN}/w_{\varrho N}$. The comparison of experimental velocities w_{ϱ} from Table 1 and $w_{\varrho N}$ from Table 2

Table 2

Material	k (W/mK)	radius, a $(10^{-3}$m)	pressure p (bar)	Nye/Nunn δ $(10^{-6}$m)	Nye/Nunn w_{tN} (mm/h)	Nye/Nunn $w_{\varrho N}$ (mm/h)	Nye/Nunn $w_{tN}/w_{\varrho N}$	Hahne/Grigull δ_1 $(10^{-6}$m)	Hahne/Grigull δ_s $(10^{-6}$m)	Hahne/Grigull w_{tl} (mm/h)	Hahne/Grigull w_{ts} (mm/h)	w_{tN}/w_{ts}	$w_{tl}/w_{\varrho N}$
Silver	419	0·25	3·48	0·570	120·24	7·20	17	0·62	0·67	46·6	56·41	2·13	6·47
Copper	385	0·36	2·41	0·665	63·72	3·60	18	0·73	0·78	24·53	30·00	2·12	6·81
		0·55	3·83	0·799	74·88	6·12	12	0·87	0·93	29·24	35·94	2·08	4·78
		0·79	5·42	0·938	83·52	7·20	12	1·02	1·10	31·81	39·06	2·14	4·42
		0·79	1·81	0·938	27·79	2·52	11	1·02	1·10	10·62	13·04	2·13	4·21
Carbon steel/Iron	50	0·19	8·42	0·345	110·88	9·00	12	0·38	0·41	44·48	55·99	1·98	4·94
		0·23	3·62	0·369	39·96	4·68	8·6	0·41	0·44	16·05	20·24	1·97	3·42
		0·23	7·24	0·369	79·92	9·72	8·2	0·41	0·44	32·10	40·48	1·97	3·30
		0·28	5·96	0·396	54·72	5·76	9·5	0·44	0·47	22·05	27·84	1·96	3·82
		0·28	2·98	0·396	27·40	2·88	9·5	0·44	0·47	11·02	13·92	1·96	3·82
Nylon/Perlon	0·25	0·21	7·65	0·136	5·04	5·04	1	0·15	0·16	2·04	2·60	1·94	0·41
		0·43	3·72	0·172	1·20	1·73	0·69	0·19	0·21	0·48	0·62	1·94	0·28
		0·43	5·58	0·172	1·80	2·59	0·69	0·19	0·21	0·73	0·93	1·94	0·28

* The resistivities of ice and water given there on page 1283 are misprinted by 10^2. For calculations correct values were used, there and here.

shows that our values are always considerably higher. For difference in radii, pressure and thermal conductivities, direct comparison is not possible, but those cases which agree sufficiently in these parameters justify the statement. As mentioned in 2·1 and 2·3 contamination in the ice always causes a decrease in wire speed, so that the higher velocities can be considered more reliable. Thus the great discrepancies between theory and experiment w_{tN}/w_{eN} are caused by both too high theoretical values and too low experimental.

5. CONCLUSIONS

1. The classical experiment for the regelation phenomenon does not only exhibit a pressure dependence but also a strong influence of the thermal conductivity of the penetrating wire.
2. The dependence of penetration rate on a surface effect of the material–ice combination could not be confirmed.
3. The big scatter in the penetration rate of metallic wires is due to an irregular water layer thickness around the wire and irregular refreezing behind the wire. For Perlon, the scatter is much smaller because water layer thickness is irrelevant.
4. A simplified model for the heat conduction mechanism yields equations for the penetration speed with relatively good agreement to experiments for metallic wires.
5. The deviations in theoretical and experimental speed for Perlon wires reveal an unseizeable defect in theoretical heat balance for heat may be supplied from melted grain-boundaries rather than from the regelated zone.

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LE REGEL DE LA GLACE. UN PROBLÈME DE CONDUCTION THERMIQUE

Résumé—Dans l'expérience classique du regel de la glace, un fil tendu passe à travers un bloc de glace, mais celui-ci-ne peut être coupé car le plan de séparation gèle à nouveau.

L'explication usuelle est seulement basée sur l'anomalie de l'eau, spécifiquement la décroissance de la température de fusion avec un accroissement de pression comme l'établit l'équation de Clausius-Clapeyron. On montre ici que la conduction thermique joue un rôle prédominant dans cette expérience.

L'expérience classique est répétée avec une variété de fils de conductivités thermiques et de diamètres différents et en appliquant des pressions variées.

A partir d'un modèle simplifié pour le mécanisme de conduction thermique on peut obtenir une équation par l'estimation de la vitesse de pénétration. Comparé à un autre travail, l'accord de cette équation théorique avec les résultats expérimentaux est très bon. Pour des fils métalliques, l'hypothèse d'une distribution linéaire de température autour du fil est meilleure que celle d'une distribution sinusoïdale.

Des fils de perlon pénètrent plus à travers la glace que ne le prédit la théorie, ce qui indique ainsi des défauts non maîtrisés dans la théorie. L'influence interfaciale du couple matériau-glace dominant la vitesse de pénétration n'aurait pas été considérée.

DIE REGELATION DES EISES—EINE PROBLEM DER WÄRMELEITUNG

Zusammenfassung—Im klassischen Experiment für die Regelation des Eises schneidet sich eine gewichtsbelastete Drahtschlinge durch einen Eisblock ohne ihn zu zerteilen; denn die Trennfuge verschmilzt wieder. Die übliche Erklärung dafür beruht nur auf der Anomalie des Wassers, speziell der Abnahme der Schmelztemperatur bei Druckzunahme nach der Clausius-Clapeyron Gleichung. Es wird hier gezeigt, dass die Wärmeleitung eine dominierende Rolle in diesem Experiment spielt. Der klassische Versuch wurde mit einer Vielzahl von Drähten unterschiedlicher Wärmeleitfähigkeit, Durchmesser und Gewichtsbelastungen wiederholt.

Nach einem vereinfachten Modell für den Vorgang der Wärmeleitung konnte eine Beziehung zur Berechnung der Durchziehgeschwindigkeit der Drahtschlinge erhalten werden. Die Übereinstimmung dieser Beziehung mit den Versuchswerten ist im Vergleich zu anderen Arbeiten sehr gut. Die Annahme einer linearen Temperaturverteilung um den Draht liefert bei metallischen Drähten eine bessere Übereinstimmung als sinusförmige Verteilung.

Für Perlonschnüre ist die experimentelle Geschwindigkeit stets grösser als die theoretische, wodurch sich noch theoretisch unerfasste Einflüsse andeuten. Ein von der Material-Eis-Paarung ausgehender dominierender Zwischenschichteinfluss konnte nicht nachgewiesen werden.

ПОВТОРНОЕ ЗАМОРАЖИВАНИЕ ЛЬДА. ЗАДАЧА ТЕПЛОПРОВОДНОСТИ

Аннотапия—В классическом эксперименте с повторным замораживанием льда нагруженная проволочка проходит через блок льда, но не разрезает его на две отдельные части, так как плоскость разреза снова замерзает.

Обычное объяснение основывается на аномальном поведении воды, и в частности, объясняется понижением температуры плавления при увеличении давления, как это явствует из уравнения Клаузиуса-Клапейрона. В данной работе показано, что в этом эксперименте доминирующую роль играет теплопроводность.

Классический эксперимент был повторен с проволочками разного диаметра и с разной теплопроводностъю при различных давлениях.

Из упрощенной модели механизма теплопроводности может быть получено уравнение для расчёта скорости проникновения. По сравнению с другими работами получено хорошее согласование теоретических и экспериментальных результатов. Лучшее согласование результатов для металлических проволочек даёт допущение линейного, а не синусоидального распределения температуры вокруг проволочки.

В отличие от теоретических расчётов перлоновые нити проходят через лёд быстрее, что свидетельствует о недостатках теории.

Не было обнаружено заметного влияния на скорость проникновения прослойки между льдом и материалом.