# Conventionel versus Petri Net Modeling of a Transport Process in Postal Automation

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**Abstract.** In modern postal sorting centers, the singulation, address reading, transport and sortation of mail pieces is done automatically by machines which are equipped with automatic address readers. The letter sorting machine considered here, can be modeled either as a discrete event system with the help of a petri net, or as a time continuous system in terms of differential equations. The steps of modeling are illustrated, the pros and cons are discussed, and an approach for throughput control by feedback is outlined.

**Keywords**: System Modeling, Transport Process, Petri Net, Distributed Parameter System, Postal Automation

## 1 Introduction

Figure 1 shows the principal components of a modern mail piece sorting machine. With the help of a special *feeding device*, mail items are fed separately into the machine, and are passed across to constantly moving transport belts. The mail items then pass an *image scanner*, which transfers the image information to the *address reading device* (ARD). This ARD can be either an automatic address reader, based on optical character recognition, or a so-called *video coding system*, where the image is displayed on a monitor and the required sorting information is keyed in by an operator. In both cases the ARD processes the images in the sequence of arrival (i.e., FIFO, see [5]) while the mail items move at a constant speed through a *delay line*. The task of this delay line is to provide the ARD with enough time so that the address reading result is available before the mail item reaches the *sorting section* of the machine. In the sorting section the mail items are separated into different stackers or trays.

A modeling of this system is required for the design of a throughput control or for simulation studies in order to predict the performance of different configurations [3, 4]. Obviously, the system is driven by discrete events, namely

- the *feeding of a mail piece*. This is a *control input*, since this event can be triggered from outside, for example by a controller.
- the *coding of a mail piece*, i.e. the event by which the ARD finishes processing of an item and the read result becomes available. This event depends on the structure of the ARD, on the loading situation of the ARD and on the complexity of the individual image

being processed. It can be considered as a *disturbance input*, since the time of its occurrence cannot be predicted precisely.

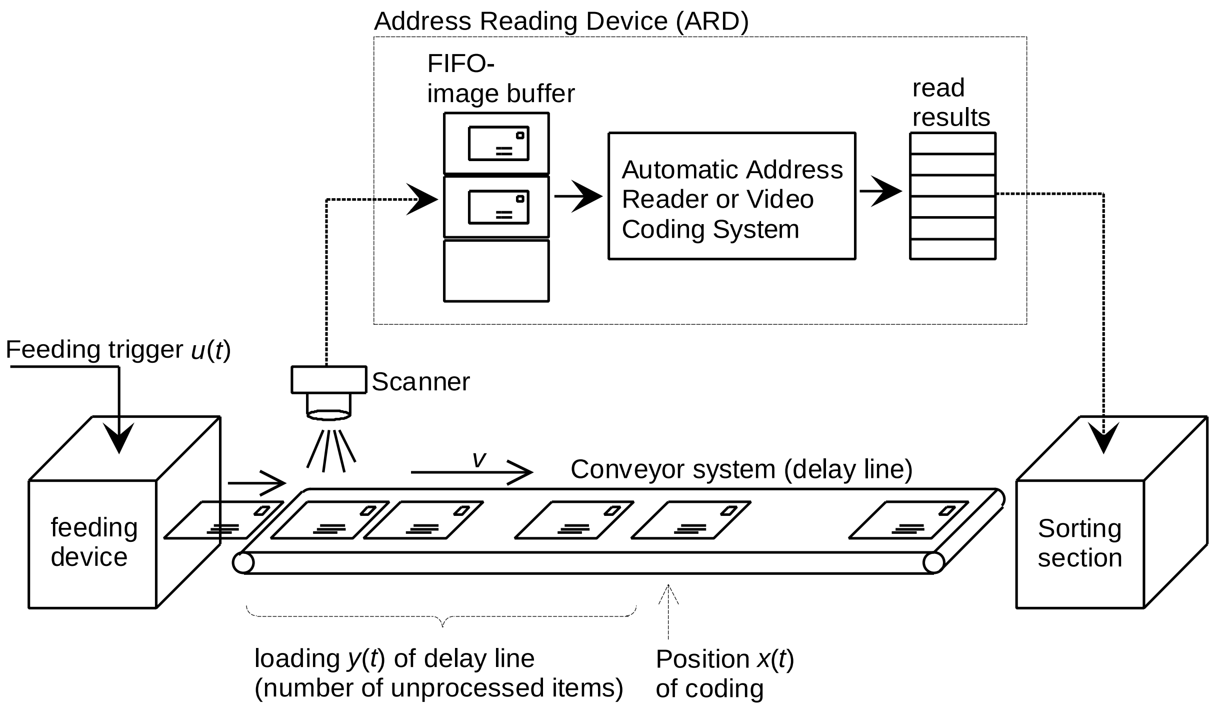


Fig. 1 Components of an automatic postal sorting machine

The system variables of interest are

- the *coding position x*, i.e. the position of a mail piece at the moment when its coding result becomes available. Sorting of the mail piece is only possible if this coding position is *left* from the sorting section (see Fig. 1).
- the *numbers of mail pieces* being in the different stages of processing, like "uncoded and within delay line", "coded and within delay line", "successfully sorted", "sorting not possible". These give the statistical information on the performance of the machine.

## 2 Modeling by a Petri Net

From Fig. 1 and the knowledge of the system, a petri net can be derived quite easily (Fig. 2, see [2] for an overview). T1 is a source transition, representing the feeding of an item to the machine. The frequency of firing can be considered a control input. Concurrent processes follow: the *image processing and address reading process* represented by P2, P3, P4, T2, T3, and the *transport process* represented by P1. The place P1 is a *timed place* which means that each marking entering P1 leaves it after a constant given time T. Depending on whether a read result is available at the end of the delay line or not, either T4 or T5 fires;

the inhibitor arc is required for this decision. This firing synchronizes the two concurrent processes and starts the sortation.

The place P3 has finite capacity $k = 1$ while all other places can be considered infinite capacity. The two inputs of the system, feeding of items and coding of items, relate to the firing of T1 and T3. The other transitions of the model fire as soon as they are enabled.

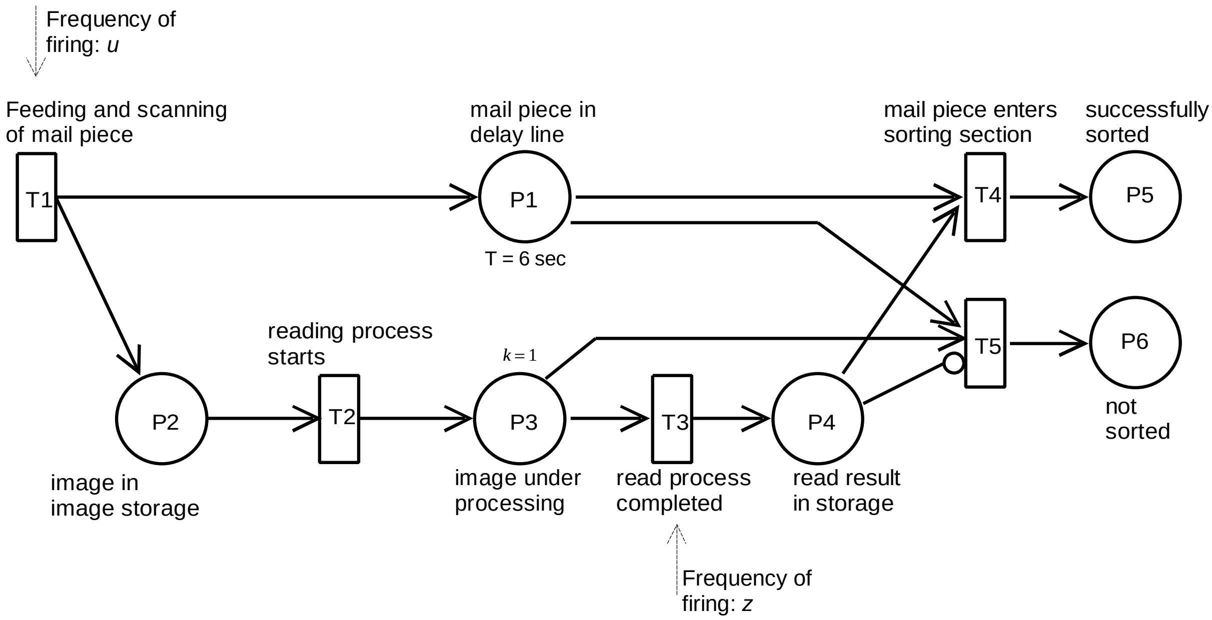


Fig. 2 Petri net model of the sorting machine

The *numbers of mail pieces* being in the different stages of processing are given by the numbers of markings in the different places.

The *coding position* $x_i$ of a mail piece *i* can be calculated from the time $t_{i,enter}$ at which T1 fires, and the time $t_{i,code}$ at which the corresponding mail piece is coded, by $x_i = v \cdot (t_{i,code} - t_{i,enter})$. Note that $x_i$ is *not* a time-continuous function, but has to be calculated for each single mail piece after it has been coded. The underlying (time-continuous) differential equation is $\dot{s}(t) = v$, and $x_i$ relates to the end of the considered time interval. This is not represented by the petri net.

From this calculation of the coding positions $x_i$ it becomes clear, that a precise simulation, including the positions of all mail pieces moving along the delay line, would require simple models of the type $\dot{s}(t) = v$ for *each* single mail piece.

## 3 Time-Continuous Modeling

If the mail pieces are considered as "dividable" so that a time continuous feeding and coding process results, then a time continuous modeling of the hole process is possible. The input "feeding" is then represented by a *feeding rate* $u(t)$ (in mail pieces per second), and the disturbance "coding" is represented by the *address reading capacity* $z(t)$ (in mail pieces per second). These two *frequencies of events*, $u$ and $z$, are also illustrated in the petri net, Fig. 2.

The most important system variable is the *position* $x$ within the delay line at which mail items switch from "unprocessed" to "processing completed", see also Fig. 1. In a time-continuous consideration of the process, $x$ is no longer related to single mail pieces, but also becomes a time-continuous variable. The velocity $\dot{x}(t)$ of this *position of coding* is

$$\dot{x}(t) = v - \frac{z(t)}{c(x,t)} \quad . \tag{1}$$

The constant transport speed $v$ is from left to right. From right to left the ARD proceeds with the *capacity* $z(t)$ (in items/sec). This capacity is divided by the concentration $c$ of mail items at $x$, which results in the corresponding velocity. The concentration $c$ is

$$c(x,t) = c(0, t - x/v) = \frac{u(t - x/v)}{v} \quad , \tag{2}$$

which is a solution of the *1st order hyperbolic partial differential equation*

$$\frac{\partial c(s,t)}{\partial t} + v\frac{\partial c(s,t)}{\partial s} = 0 \tag{3}$$

with the *boundary condition* and the *initial condition*

$$c(0,t) = \frac{u(t)}{v} \quad , \tag{4}$$

$$c(s,0) = c_0(s) \quad , \tag{5}$$

describing the distributed parameter system considered here [1, 4, 6]. The variables are

- $c(s,t)$ — the *concentration* of mail items on the transport belt (measured in pieces per meter),
- $v$ — the constant *transport speed*,
- $t$ — the *time*,

| | |
|---|---|
| $s$ | the *spatial coordinate variable*, |
| $u(t)$ | the *feeding rate* (in pieces per second); this is the control input, |
| $c_0(s)$ | the *initial covering* of the transport belt with mail items. |

With the solution (2) of equation (3), equation (1) becomes

$$\dot{x}(t) = v \cdot \left(1 - \frac{z(t)}{u(t - x/v)}\right), \tag{6}$$

which is a nonlinear differential equation with the varying delay $x/v$ in the input $u$.

Figure 3 shows the structure of the model representing eq. (6). The input $u$ affects the nonlinear block via the varying delay. From the delayed input $u(t)$ and from the address reading capacity $z(t)$ the nonlinear block produces the velocity $\dot{x}(t)$. The state variable $x$ itself defines the varying delay. The system reacts in an unstable manner, for example, on two different but constant values of the inputs $u$ and $z$.

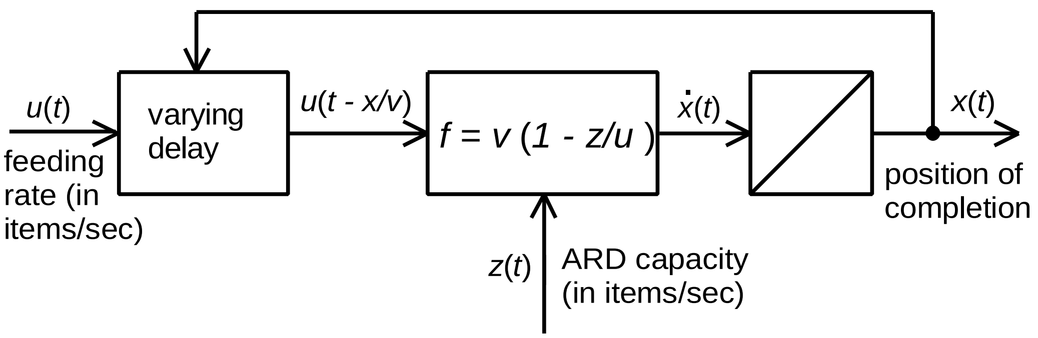


Fig. 3 Part system "coding position $x$"

Another system variable of interest is the number of mail items being in the delay line and waiting to be coded. In a time-continuous formulation this is a quantity $y$ representing the *loading of the delay line*. For $y$, the simple differential equation

$$\dot{y}(t) = u(t) - z(t) \text{ for } y \geq 0 \tag{7}$$

is obtained, see Fig 4. Also this part system behaves unstable. In [4] a switching controller is introduced which stabilizes $y$ in a time optimal way.

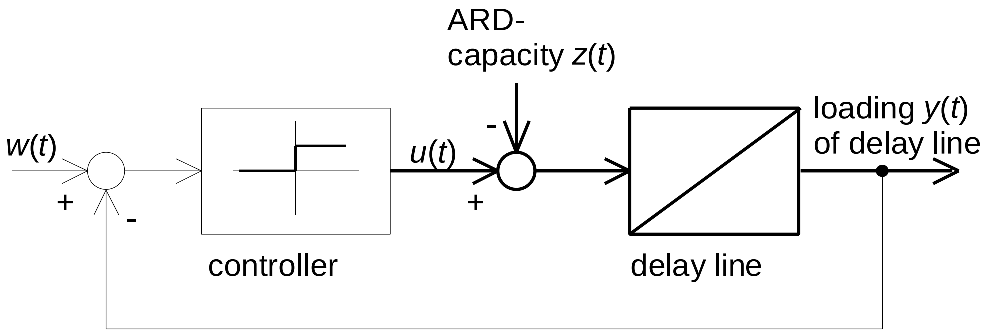


Fig. 4  Part system "loading $y$ of delay line" (thick lines) and stabilizing feedback control (thin lines)

## 4 Discussion

Although the technical process considered here is dominated by discrete events, a purely time continuous modeling is possible, since an equivalent continuous interpretation of the process and its input and output variables exists. Nevertheless, the models differ significantly, not only in representation, but also in the usefulness for different tasks like controller design, representation of special situations, and simulation.

*1. System insight and representation of system variables:* The *position x* of coding, and an important characteristic of the model, the *varying delay*, are represented in the *continuous* model, but not in the petri net. The reason for this is that the movement of mail items along the delay line is in fact a continuous process. For the representation of such processes, the petri net would need to be extended in a cumbersome manner. However, the petri net is superior in representing the number of items in the different stages of the process: The number of markings in the different places directly gives the required information. The *unstable behavior* of the plant can be analyzed by the petri net as well as by the time continuous model. For instance, if the ARD capacity $z$ is small compared to the feeding capacity $u$, then, in the petri net the number of markings in P2 will exceed any limits; in the continuous model, the outputs of the integrators in figures 3 and 4 will exceed any limits.

*2. Controller Design:* The *design of a stabilizing controller* is possible based on either models: the most simple approach based on the petri net would be, to give P2 a limited capacity $k$, i.e. the control rule would be: "Feed a mail piece as soon as the number of mail pieces waiting for ARD processing falls short of the value of $k$". A corresponding time-continuous control law can easily be found [4], and is illustrated in fig. 4. However, the design of a controller which stabilizes the system *and* keeps $x$ within certain limits of the delay line has only been successful based on the continuous model: the idea presented in [3, 4] is to implement a disturbance feedback by measuring the disturbance $z$ and by choosing the input $w(t)$ of the stabilizing control loop (fig.4) to $w(t) = kz(t)$. In the petri net this would mean to vary the finite capacity $k$ of the place P2 depending on the frequency $z$ of firing of T3, which is not an obvious approach.

*3. Special situations*: The continuous model does not correctly represent the fact that $y$ cannot become smaller than 0. The petri net does: the corresponding place runs empty and the subsequent transitions are disabled (which enforces $z = 0$). Furthermore, the continuous model cannot represent the feature that the ARD process is interrupted if the corresponding mail item leaves the delay line without read result (i.e., the firing of T5 in Fig. 2 is not represented in the continuous model).

*4. Simulation*: For the simulation of the model, with or without controllers, a *hybrid* representation is adequate. It includes the petri net as shown in Fig. 2 with an added time continuous layer describing the movement of each single mail piece through the delay line, see Fig. 5. This presupposes that the markings can be distinguished (*colored* petri net). At *SIEMENS Electrocom* in Konstanz, Germany, the system model, together with different controllers, has been simulated extensively based on this hybrid approach, before the final control algorithm was implemented in the field [4].

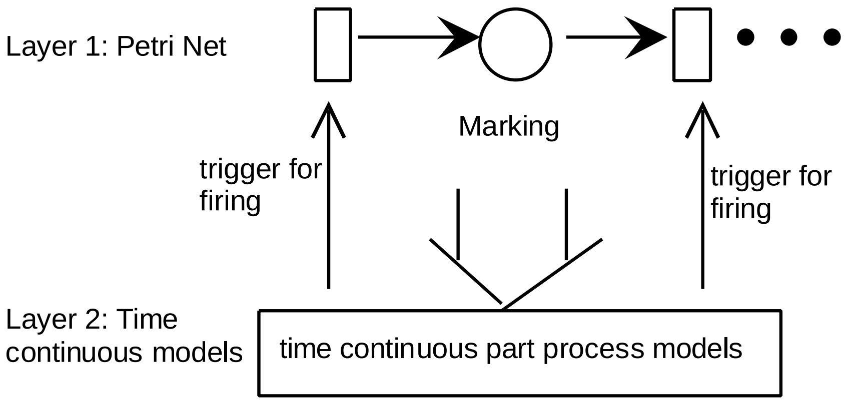


Fig. 5: Hybrid modeling in two layers

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