Bachelor Thesis

Application of biomechanical numerical analysis on experimental data

Final Presentation

Mariella Kast

February 05, 2016

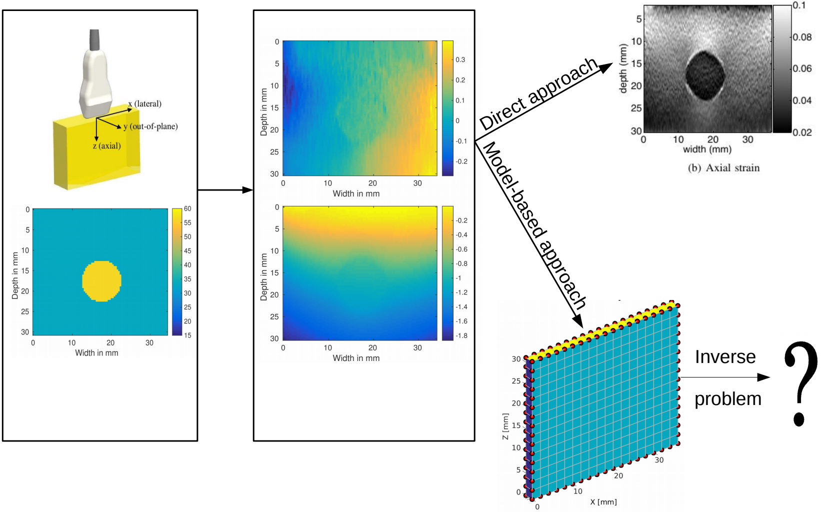
Problem description

Overview of the elastography procedure

1) Image acquisition

2) Displacement map

3) Material parameter distribution



Inverse problem

- Cost Function:

$$F(\boldsymbol{\theta}) = \frac{1}{2} \parallel (\boldsymbol{u} - \hat{\boldsymbol{u}}) \parallel^2 + F_{\text{reg}} = \frac{1}{2} \sum_{i=1}^{N_{\boldsymbol{u}}} (u_i - \hat{u}_i)^2 + F_{\text{reg}}$$

- Regularization:

$$F_{\text{reg}} = \frac{w}{2} \sum_{i=1}^{N_e} \sum_{j=1}^{N^i_{\text{neig}}} (\theta_i - \theta_j)^2$$

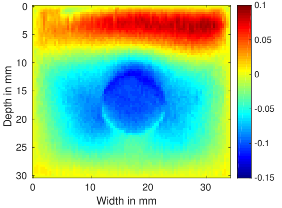
- Optimization: *BFGS* algorithm calling *FEBio* for evaluation of F and its derivatives with respect to the material parameters $\boldsymbol{\theta}$.

Model assumptions

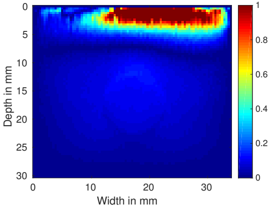
- Plane strain with negligible movement in the out-of-plane direction
- Gaussian distributed error on data
- An isotropic/Mooney-Rivlin material model can sufficiently represent tissue behavior
- Displacement values at the boundary of the ROI are reliable enough to be used as Dirichlet Boundary Conditions
- Downsampling data without loss of information is possible

Forward Analysis

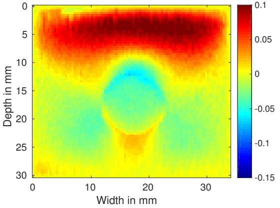
Differences $\boldsymbol{d}$ between measured and predicted axial displacements



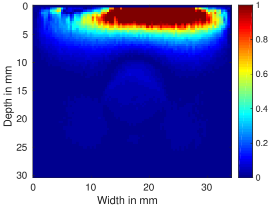
(a) Linear-elastic material: $\boldsymbol{d}$



(b) Linear-elastic material: $\boldsymbol{d}_{\text{rel}}$

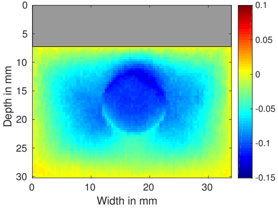


(c) Mooney-Rivlin material: $\boldsymbol{d}$

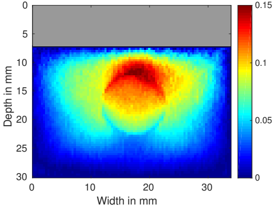


(d) Mooney-Rivlin material: $\boldsymbol{d}_{\text{rel}}$

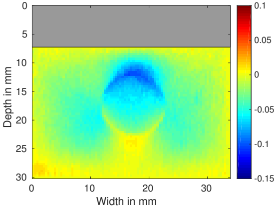
Differences $\boldsymbol{d}$ between measured and predicted axial displacements



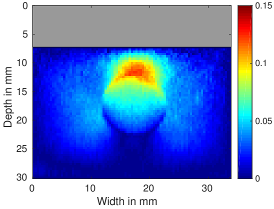
(a) Linear-elastic material: $\boldsymbol{d}$



(b) Linear-elastic material: $\boldsymbol{d}_{\text{rel}}$

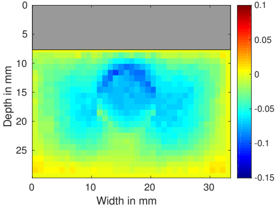


(c) Mooney-Rivlin material: $\boldsymbol{d}$

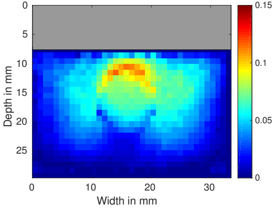


(d) Mooney-Rivlin material: $\boldsymbol{d}_{\text{rel}}$

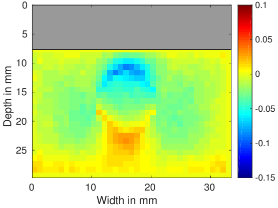
Differences between a spherical inclusion and the experimental displacements



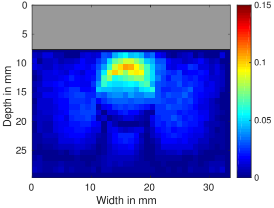
(a) **Linear-S**: $\boldsymbol{d}$



(b) **Linear-S**: $\boldsymbol{d}_{\mathrm{rel}}$

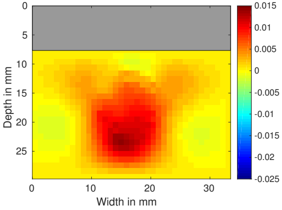


(c) **Mooney-S**: $\boldsymbol{d}$

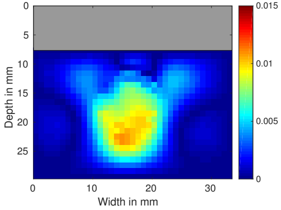


(d) **Mooney-S**: $\boldsymbol{d}_{\mathrm{rel}}$

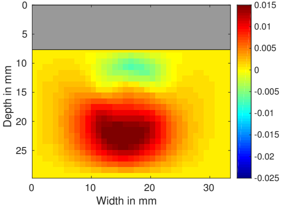
Differences between a cylinder and a sphere



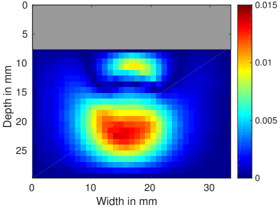
(a) **Linear-S**: $\boldsymbol{d}$



(b) **Linear-S**: $\boldsymbol{d}_{\mathrm{rel}}$



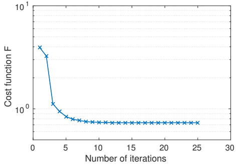
(c) **Mooney-S**: $\boldsymbol{d}$



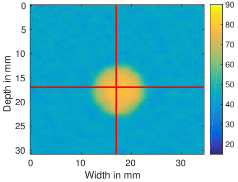
(d) **Mooney-S**: $\boldsymbol{d}_{\mathrm{rel}}$

Inverse Analysis

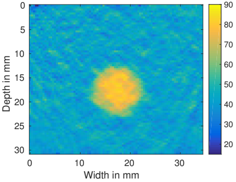
Artificial example



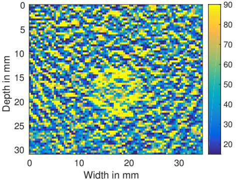
(a) Convergence, SNR=40 dB, $w = 10^{-5}$



(b) Young Modulus (kPa), SNR=40 dB, $w = 10^{-5}$

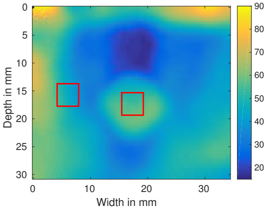


(c) Young Modulus (kPa), SNR=30 dB, $w = 10^{-5}$

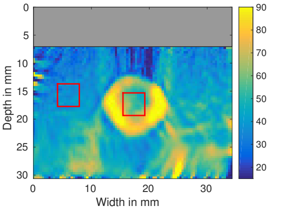


(d) Young Modulus (kPa), SNR=30 dB, $w = 0$

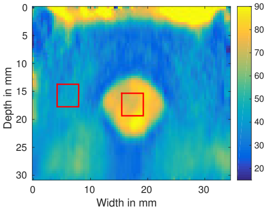
Optimal Material Parameter Distribution $\boldsymbol{\theta}$



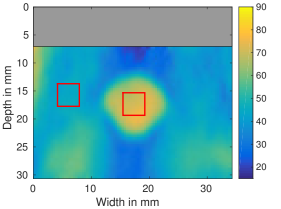
(a) **Linear-N**, $w = 10^{-5}$



(b) **Linear-S**, $w = 10^{-7}$

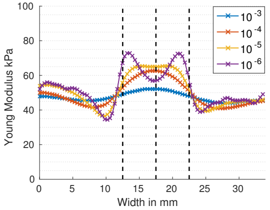


(c) **Mooney-N**, $w = 10^{-5}$

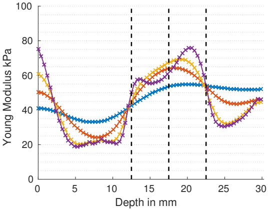


(d) **Mooney-S**, $w = 10^{-4}$

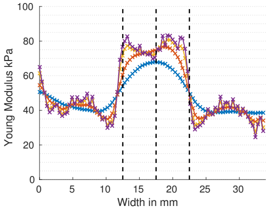
Regularization on the normal domain



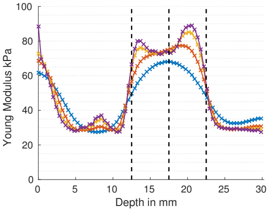
(a) **Linear-N**, θ in x-direction



(b) **Linear-N**, θ in z-direction

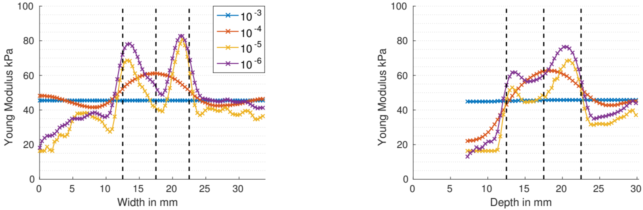


(c) **Mooney-N**, θ in x-direction

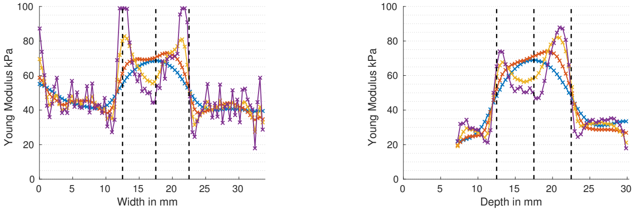


(d) **Mooney-N**, θ in z-direction

Regularization on the smaller domain

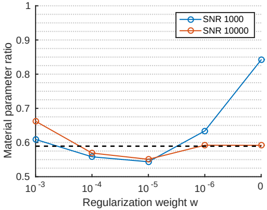


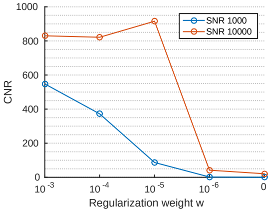
(a) **Linear-N**, θ in x-direction (b) **Linear-N**, θ in z-direction



(c) **Mooney-N**, θ in x-direction (d) **Mooney-N**, θ in z-direction

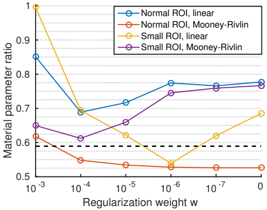
Contrast analysis

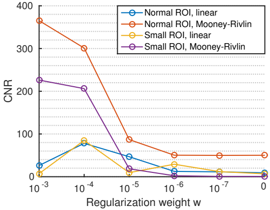




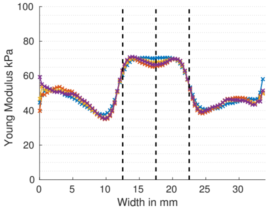
(a) Ratio of mean Young Moduli $E_{\text{rat}} = \frac{E_{\text{background}}}{E_{\text{target}}}$

(b) Contrast to Noise Ratio, $\text{CNR} = \sqrt{\frac{2(\mu_a - \mu_b)}{\sigma_a^2 + \sigma_b^2}}$

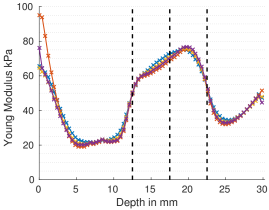




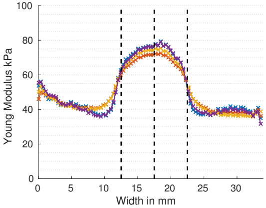
Noise on Boundary Conditions: SNR 1000



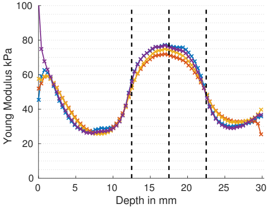
(a) **Linear-N**, θ in x-direction



(b) **Linear-N**, θ in z-direction

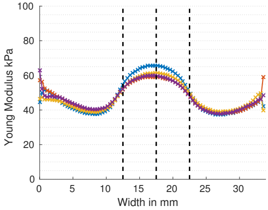


(c) **Mooney-N**, θ in x-direction

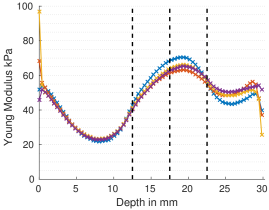


(d) **Mooney-N**, θ in z-direction

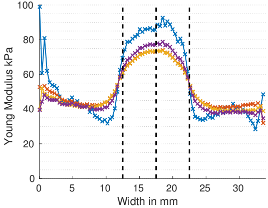
Noise on Boundary Conditions: SNR 100



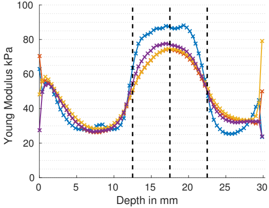
(a) **Linear-N**, $\boldsymbol{\theta}$ in x-direction



(b) **Linear-N**, $\boldsymbol{\theta}$ in z-direction



(c) **Mooney-N**, $\boldsymbol{\theta}$ in x-direction



(d) **Mooney-N**, $\boldsymbol{\theta}$ in z-direction

Uncertainty Quantification

Sampling Algorithm

Algorithm 1 Metropolis sampling strategy for the inverse problem

0) Initialize a guess $\boldsymbol{\theta}_0$ for the material parameters and evaluate the cost function $F(\boldsymbol{\theta}_0)$ in a forward analysis (e.g. with *FEBio*)

for k=1:N (Number of samples) **do**

1) Draw a new candidate sample $\boldsymbol{\theta}_c \sim \mathcal{N}(\boldsymbol{\theta}_k, \boldsymbol{\sigma}_{\mathrm{s}}^{\mathbf{2}})$ and assess the value of $F(\boldsymbol{\theta}_c)$

2) Calculate the acceptance ratio $a = \exp(-\frac{1}{\sigma_{\mathrm{n}}^2}(F(\boldsymbol{\theta}_c) - F(\boldsymbol{\theta}_k))$

3) Accept the candidate $\boldsymbol{\theta}_c$ with probability $A(\boldsymbol{\theta}_k, \boldsymbol{\theta}_c) = \min(1, a)$:

if $\boldsymbol{\theta}_c$ is accepted **then**

$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_c$

else

$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k$

end if

end for

Formulation for fewer parameters

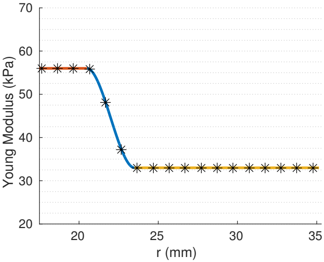


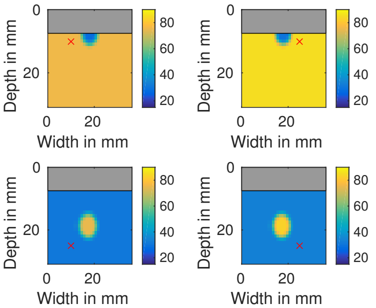
Figure: Function θ and its FEM-discretization $\boldsymbol{\theta}$ (black markers) for the parameter set: $r_0 = 3,\ x_0 = 17.5,\ z_0 = 17.5,\ E_\mathrm{i} = 56,$ $E_0 = 33,\ w = 3$

$$\theta = \left\{ \begin{array}{ll} E_\mathrm{i}, & r < r_0 \\ f(r - r_0), & r_0 < r < r_0 + w \\ E_\mathrm{o}, & r > r_0 + w \end{array} \right\},$$

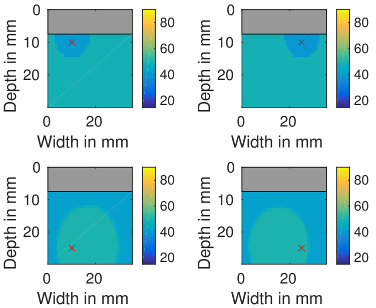
$$f(t) = E_\mathrm{i} - \frac{3t^2(E_\mathrm{i} - E_\mathrm{o})}{w^2} + \frac{2t^3(E_\mathrm{i} - E_\mathrm{o})}{w^3}$$

$$r = \sqrt{(x_c - x_0)^2 + (z_c - z_0)^2}$$

Solution for reduced parameters



(a) Young Modulus (kPa), Sampling



(b) Young Modulus (kPa), deterministic

Comparison: Reduced - all material parameters

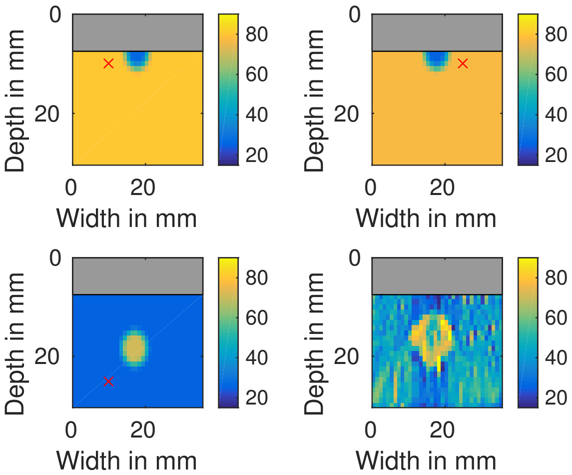


Figure: Young Modulus (kPa)

Position of the Circle

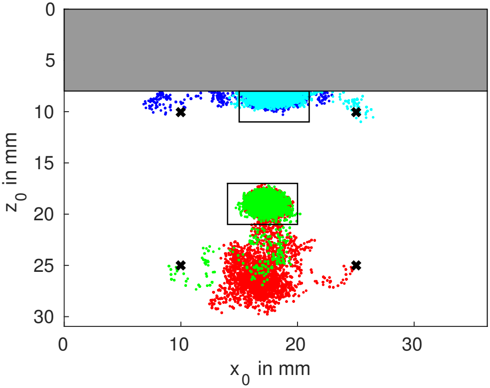
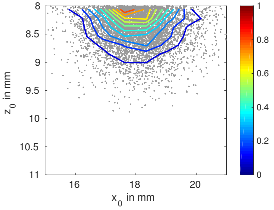
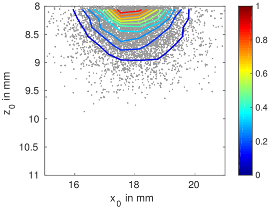


Figure: 100000 data points (including burn-in) for the center of the circle sampled by the Metropolis algorithm. Simulations are conducted on the case **Mooney-S** with a resolution of (36×22) elements.

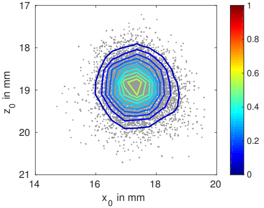
Position of the circle: Zoom in



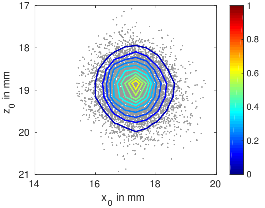
(a) Starting guess (10,10)



(b) Starting guess (25,10)

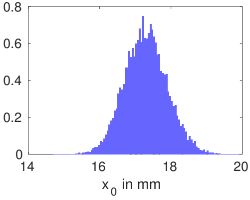


(c) Starting guess (10,25)

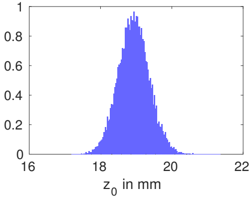


(d) Starting guess (25,25)

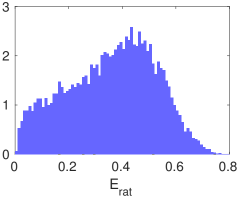
Histograms for starting guess (10,25)



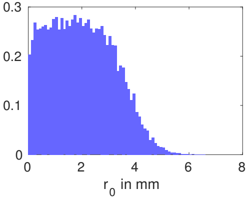
(a) PDF of x_0



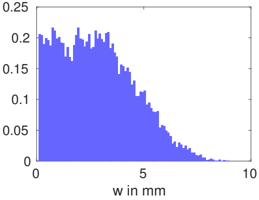
(b) PDF of z_0



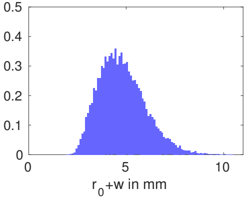
(c) PDF of E_{rat}



(d) PDF of r_0

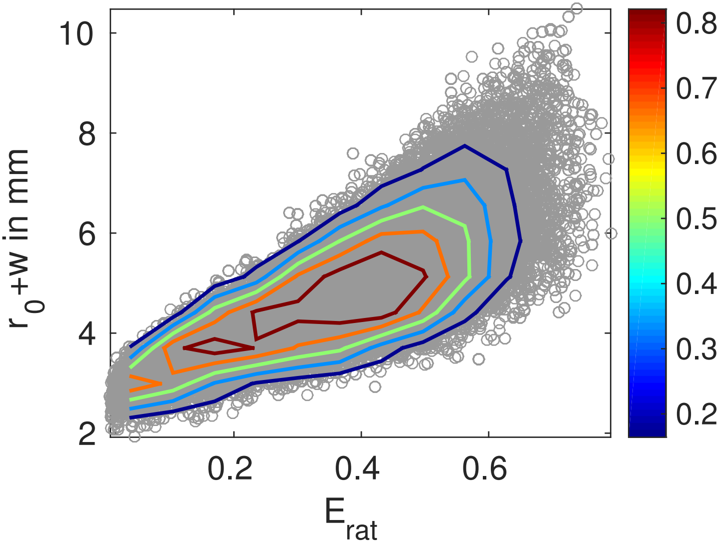


(e) PDF of w

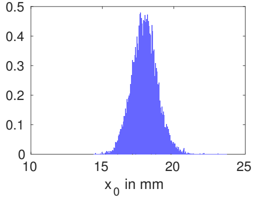


(f) PDF of $r_0 + w$

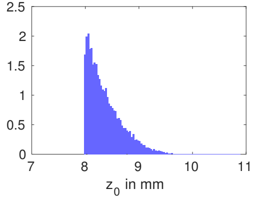
Correlation between stiffness and size of the circle



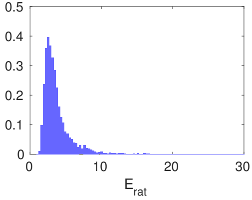
Histograms for starting guess (10,10)



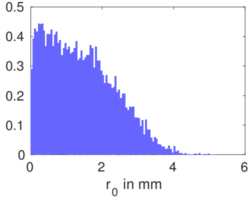
(a) PDF of x_0



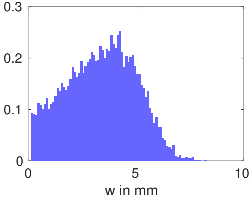
(b) PDF of z_0



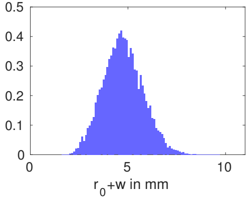
(c) PDF of E_{rat}



(d) PDF of r_0



(e) PDF of w



(f) PDF of $r_0 + w$

Outlook

Open Questions

- Suitable material model
- Effects of ultrasound
- More complex geometries (and in this context, the assumption of plane strain)
- Non-uniqueness of the problem due to Dirichlet Boundary Conditions

Ideas for improvement

- Veronda-Westermann material or consideration of error in the material model
- More complex model for uncertainty approach or UQ for every Finite Element
- More efficient sampling algorithm (Gibbs, MALA)
- Extended use of priors